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THE MATHEMATICS TEACHER

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Edited by William David Reeve

What Are the Mathematical Needs of the High School Student?*

By James H. Zant

Oklahoma A. & M. College, Stillwater, Oklahoma

In deciding what mathematics should be taught at any grade level it is necessary to know the mathematical needs of the students. However, unless we subscribe to the doctrine that no one should learn more about anything than he will need to live at that particular age level, we must also consider the future needs of the students as well. I do not subscribe to this doctrine, hence in this discussion considerable weight will be given to the mathematical needs of adults in deciding what shall be taught to children. This seems especially necessary at the high school level, since, in the case of mathematics, the student is often taking his last course in the subject.

The problem will be discussed from the standpoints of various types of needs, those which are required by every well educated man or woman to live a normal and useful life as an ordinary citizen, and those which are required in special jobs, professions, etc., as well as in getting pleasure and enjoyment out of life.

1. *Mathematics Needed by the Ordinary Citizen.* This is fairly well agreed on by most school people. It is essentially the content of mathematics through the eighth grade. The mathematical aims for this period should include provisions for developing a genuine understanding and appreciation of the fundamental operations of addition, subtraction, multiplication, and division with whole numbers, decimals, and fractions, some idea of the principles and modes of thinking so well illustrated by mathematics, a substantial introduction to the mathematics of every day life and a dependable foundation for life needs and for subsequent courses in mathematics. While emphasis will be placed on meanings and understandings, close attention must be paid to skills and knowledge. The Post-War Planning Commission has called this *Functional Competence.* "The school shall guarantee functional competence in mathematics to all who can possibly achieve it."[1] The Commission made no attempt to define "Functional Competence" explicitly. As mathematics teachers we have long known that students are leaving the high school deficient in the basic skills and knowledge of arithmetic and other phases of elementary mathematics. We believe that it

* Read on October 15, 1948 before the Northwestern Section of the Oklahoma Education Association at Alva, Oklahoma.

[1] The Second Report of the Commission on Post-War Plans, The Mathematics Teacher, Vol. 38, No. 5, pp. 196.

is one of the functions of the high school teacher of mathematics to do something about it. The task of seeing that the student has a knowledge of mathematics adequate for every day life falls peculiarily on the high school, since the student is there last. Important items of knowledge or skills which have been forgotten or never learned must be retaught during the high school years.

Though "functional competence" was not explicitly defined, a fairly definite basis for it was suggested by the Commission on Post-War Plans. It is clear that modern technology has stepped up the minimum requirements in mathematics for effective citizenship and that it is already a big step higher than the fundamental operations of arithmetic. Training in high school mathematics has become a valuable asset to a young person in industry or business as it was to many boys in the armed forces during the recent war.

Hence the Commission suggested a check list which was not based on any extensive research and which will probably change as conditions of life change in our country. This check list appears in the Second Report of the Commission mentioned above[2] and includes such items as the fundamental operations with whole numbers, fractions, decimals, and percents an understanding of ratio, ability to use tables, simple statistics, understanding the use of geometric ideas like angles, polygons, solids, etc., measurement and measurement devices, the meaning and use of formulas, signed numbers, equations, similar triangles, the meaning and use of mathematics of the home, community and business, and the like.

The content of this material is not so important perhaps as the idea that it should be the responsibility of every teacher of mathematics from grades 1 to 12 to see that the student has this competence before he leaves the high school. Much of this will be accomplished in the earlier grades and perhaps a course in general mathematics in the high school should be expected to complete the task. However, it is well known that many high school graduates are not acquainted with all of these items. The traditional sequential courses in high school mathematics, that is, algebra, geometry and trigonometry, as now taught, do not insure these skills, Hence, it may be necessary to reteach some of these things; it will certainly be necessary to see that they are known before the student leaves the secondary school. The time has passed for "passing the buck" to the teacher of some lower grade and excusing ourselves by saying that the student should have learned to add or to solve simple equations before he got to our class. It is everybody's responsibility.

Therefore, at the Oklahoma State Department of Education Work Shop at Stillwater in the summer of 1948 we tried to set up courses in 7th and 8th grade mathematics and in General Mathematics that would insure the student learning the things needed for every day living. This was done without reference to any particular textbook, but with the idea of trying to meet the pupils' needs. However, there is not complete agreement by any means among textbook writers or others as to just what these needs are. This, with the practical situation of the teacher having to use some textbook, influenced us to suggest a rather wide range of topics or units from which a course in general mathematics may be selected. As the needs of students in particular areas are more definitely determined or after needs change it will be necessary to make changes in the units selected as well as in the units themselves. Such development is expected as time passes.

Again, it should be emphasized here that the sequential courses in mathematics do not necessarily accomplish this result. We might say that a student who has had the sequence algebra, geometry, advanced algebra, trigonometry, etc., and who is competent in the areas mentioned

[2] *Ibid.*, pp. 197–198.

above probably learned these things before he took the other courses and has not forgotten them.

Hence, the custom prevalent in many of our high schools of putting the bright students in the algebra class merely because they can learn the traditional courses may not be the best for them or for the school. Such practice tends to cut down the efficiency and standards of the general mathematics course and the sequential courses may not fit the needs of the student unless he expects to study more advanced mathematics or some of the subjects which demand such a knowledge. In other words, this does not solve the problem. It may merely penalize the bright student simply because he has more than the average intelligence. The thing to do is to strengthen the general mathematics course so that it will actually meet the needs of the non-mathematical non-scientific student and to point out to them and to their parents that the course has different purposes or goals. The course should not be for "dumb bells" and, while it may not be as rigorously difficult as algebra and geometry, it can, if it is well organized and administered, be just as challenging and for many students much more useful.

2. *The Mathematics Needed by Specialized Groups.* This has a wider and more extended range but actually calls for little fundamental reorganization of the present courses which we have called "the sequential courses in mathematics." The reason for this is that most of the specialized groups we have must study mathematics at least through the high school level and many of them must go beyond that stage. We shall say then that the high school has the function "to provide sound mathematical training for our future leaders in mathematics, science, technology and other learned fields" as well as sound mathematical training for a growing number of young people who will enter semi-professional fields in industry and business which, while they do not demand advanced training in mathematics, do demand a thorough understanding of the elementary principles of the subject and definite skills not included in the check list discussed above.

The need for such trained men and women, especially for those with various degrees of college training in science, mathematics, and technology, is still critical. Policies in effect during the recent war resulted in a drastic depletion of the scientific manpower resources of the nation. Not only were well trained young scientists and technologists used up in combat divisions, but, since few were allowed to begin and continue their training, the sources of such personnel were all but dried up. While a vast number of young men are now being trained for such work, scientific discoveries and improvement in industry and manufacturing as well as an increasing demand of the national defense program make it hard to meet the need.

Training of scientific personnel may well be definitely hampered by the national policies and necessities regarding drafting men for the armed forces. It is important that provisions be made to insure proper training for those young men who are capable of doing outstanding work in science, mathematics and technology. They should be encouraged to begin their training before they have to enter military service and it is the obligation of the government to see that they are able to continue it after they leave the service. The high school's part in this program is to see that they have thorough training in elementary mathematics and science, much more thorough than they have had in the past, and that they are encouraged to continue this training into college.

The mathematics courses and training for this group of students should probably be the regular sequential courses already taught in high school, algebra, geometry and trigonometry, extended if necessary to cover four years, or general mathematics courses which cover the same

ground. It is not probable that they will learn too much mathematics. Every attempt should be made to organize and to teach the courses better but the need is for more mathematics rather than less.

The needs for the group who will enter the technical semi-professional field or business or industry either at the end of the high school period or after short training courses are quite varied and it is not possible to state briefly or even to know with much certainty just what knowledge will be useful to them. This is true because needs vary with the community and type of work and with the individual. However, most of this work will call for a knowledge of algebra, geometry, advanced algebra and in many cases trigonometry. Hence these courses should be required, but topics in general mathematics like elementary statistics, the mathematics of compound interest, annuities, investments, insurance and other parts of the mathematics of finance will be useful to some groups. To determine these it will be necessary to study the needs of the community, the type of work the students will enter, the types of schools or other training they will undertake, etc. There is no set rule or set of courses which will fit all students. However, it is certain that a basic ground work in algebra will be necessary for practically all groups of this sort and many will be able to profit by more algebra and perhaps geometry and trigonometry.

3. *Mathematics Needed by Future Teachers of Mathematics.* In a group of this sort, where most of us are teachers of mathematics or science, we should not neglect to discuss the needs of our own group and the students whom we hope to encourage to come and help us. It is still true that the majority of students who study mathematics in the colleges and universities are either teachers or will go into teaching. The mathematics needed in high school by this group is again "as much as you can get" but we should also give advice and encouragement to get the proper kind of people into the teaching of mathematics. They should observe while they are in high school the type of work a high school teacher of mathematics has to do and realize that they will probably teach the sequential courses through trigonometry as well as the general mathematics courses and perhaps courses for some of the specialized groups.

We can tell them what they should take in college to prepare for the various types of teaching they may be called on to do. Unfortunately we would also have to tell them that courses preparing them for this work may not be available in the colleges. I mean by this that most college departments of mathematics give the traditional courses in mathematics as we have done for many years. These courses prepare a prospective teacher to go into the high school and teach the standard sequential courses there fairly well. We assume, perhaps falsely, that enough college mathematics will enable the teacher to go into a high school or junior college situation and organize the proper sort of a general mathematics course to fit the needs of the community and children or to organize and teach a course to a group who expect to enter some industry or business organization in the community. This problem must be given attention by both the college departments and by the high school and junior college teachers.

This point should be emphasized. Various groups need mathematics in various ways. However, knowledge which is needed must be learned accurately and in such a way that it can be retained until needed. Our one worst fault has been a poor knowledge of what the student needs to learn. People who leave our schools at whatever level should know the mathematics needed for their future work or jobs and it is our obligation (all of us) to see that they know it accurately and well. The needs are not definite and unchanging and we must keep in mind that it is always possible to improve our knowledge of these needs and of ways of teaching the subject matter to students.

Is Mathematics Out of This World?

By Albert W. Recht
University of Denver, Denver, Colorado

When we were teaching air crew mathematics during the war, I was interested in the response of students to this problem: "The temperature is now 7° above zero; yesterday at this time the temperature was 5° below zero. How many degrees warmer is it today than it was yesterday?" Invariably students would give the correct answer, 12°. Yet when you asked them to subtract -5 from $+7$ there was a great variety of answers, perhaps because they were trying to recall a rule about changing the sign (which sign?) and adding.

Students will solve problems better if they are kept in the realm of common sense rather than in the unreal world of formal rules. Instead of using the rule of changing the sign and adding, wouldn't it be more logical always to think what number must be added to -5 to bring it up to a $+7$? Another example, when subtracting -8 from -12, just think what number needs to be added to -8 to produce -12. In this connection I should like to mention the consistency in Britton and Snively's *Algebra for College Students* with which simplification of equations is handled completely throughout the text. Without exception, instead of using the rule of transposition and changing of signs, the common-sense method of adding the same quantity to both sides of the equation is used, even when the most complicated operations in college algebra are involved. This seems a minor point, but it emphasizes my plea for keeping mathematics in the realm of reality, instead of in a never-never land of make-believe.

If you ask for the total number of raspberries in nine boxes each of which contains no raspberries, you always get the right result—no raspberries. Yet a request for the product of 9 by zero often brings forth the answer 9, even in college, or should I say, especially in college. I have had almost half of a college class make this kind of response to a similar type of question. But I have never been able to catch a third-grader napping; probably because the third-grader is still living in the land of reality—the land of no raspberries.

A student in a trigonometry class gets 600 feet for the height of a flagpole and goes blissfully on to the next problem without even realizing that flagpoles don't grow that high, and that the answer is probably 60 feet—another case of the misplaced decimal point. Isn't it odd that students spend so much of their time in arithmetic without even learning to estimate correctly? And yet the operation of estimation is one of those that occurs most often in real life, and is one that can be used frequently as a check. One can illustrate here by using the example of the boy who is taking his best girl into Murphy's Restaurant and who takes a furtive look at his pocket-book and makes a quick estimate of the depth to which he may go financially.

In arithmetic if you say the total of two numbers is 15 and one of them is 9, they will always tell you that the other is 6. But in algebra, in the land of make-believe, if the sum of two numbers is 100, and one of them is x, as often as not they will tell you that the other is $x-100$. Of course, by just looking at it you can't easily see that x is a smaller number!

In one of my engineering classes several of the students got the semi-annual rate of interest, $r/2=1.8\%$, yet found the annual rate to be $r=0.9\%$. If they had been taught by the Britton and Snively method of always multiplying both sides of the equation by the same number they would not have come out with such a senseless answer.

A student labels his answer cu. in. when it ought to be lb./sq. in. because he has acquired the out-of-this-world detachment that often comes from constant solution of out-of-this-world exercises. Many more examples of this sort of loose and listless thinking could be given.

May I add one of my greatest gripes. We often dwell at great length in algebra on the solution of quadratic equations by factoring to the exclusion of the methods of completing the square and the use of the quadratic formula in the approximation of the roots. We even build up complicated systems of factoring and handling of algebraic equations in preparation for this great event of solving quadratics by factoring. Yet one authority from the world of reality says that if there is one thing you can be sure of it is this: whenever you encounter a quadratic equation in real life, it won't factor. Why build up a complicated system of mathematics in an unreal world of the classroom when the whole field of stern usable mathematics is available?

Is it any wonder that students give us thoughtless and careless solutions of problems? It seems to me that we in mathematics have only ourselves to blame. If we insist on having our students live in a world of unreality, we are likely to have them come out with fantastic results. The mathematics we teach must start from real and interesting situations if we are to expect our students to be sensible about it.

In this connection may I list some wild ideas I have. Why not rewrite texts in mathematics in such a way so that the application of the mathematics is always given first emphasis. Of course I realize that no mathematics can be fully practical as long as it is taught in a classroom. The proper place to learn machine-shop mathematics is in a machine shop or an industrial engineer's drafting room.

However, you can duplicate some of the elements of real life. The Link trainer looks as if it might fly out of the window. When you see the machine in operation and watch a gadget crawl over the surface of a table duplicating the student's attempt to make a correct entrance into, let's say, the Salt Lake City airport, and when you hear the instructor tell the student by telephone that the first attempt was wrong and that he will have to come in again, you will begin to realize how near to practicality this device comes.

And when you read of the Link Navigational Trainer, a steel skeleton framework which holds a plane's cockpit and has room for a pilot and a navigator to make imaginary flights by taking real sights on imaginary stars shining overhead in a precision-designed sky, you can understand again how close to reality things can come. The machine has the advantage that here a navigator can fly his crew to destruction without harming a hair of their heads, or is this a disadvantage? You can come as near to practical mathematics as you can approach the duplication of the situation in which that mathematics arises. That's why elementary schools do such excellent work when they duplicate the bank, the department store with its sales-slips, and the vegetable wagon with its prices.

So why not take our mathematics texts and humanize their contents. For example, here are the titles of chapters in a typical college algebra:

1. Rectangular Coordinates and Graphs.
2. Systems of Linear Equations.
3. Ratio, Proportion, and Variation.
4. Computation and Logarithms.
5. Progressions.
6. Permutations and Combinations; Probability.
7. Infinite Series.

These titles come from a world of college mathematics, not a world of reality. Why couldn't these chapter headings be instead?

1. Dow-Jones Averages and Cycles of Depression and Prosperity.
2. Telling Time by the Stars.
3. How Physics Applies to Life.
4. Multiplication Made Easy—How to

Use the Slide Rule.

5. Installment Buying, FHA Loans, and Mortgages.
6. Statistical Methods, Marking on the Curve. Monte Carlo: The House Always Wins.
7. How Mathematical Tables are Made.

Or from a Trig text:

1. Solution of Right Triangles.
2. Trigonometric Properties of Angles.
3. Oblique Triangles.
4. Trigonometric Equations.

Why not instead?

1. How High is that Flagpole?
   How Wide is that River?
   Laying out Highway Curves.
2. The Pendulum Swings. Simple Harmonic Motion.
3. Land Surveying.
4. The Paths of Comets and Planets.

Most mathematics teachers are horrified at the idea of giving up the classical traditional mathematics. They should be more horrified at the loss of popularity of mathematics courses in our high schools and colleges at a time when the world is pretty well convinced that it is mathematics and science that make the world go round.

My opinion is that the only way to prevent this trend away from mathematics is to put mathematics back into this world for most people. Is there any validity in the idea that culture in mathematics can be obtained best by studying classical fields which are seldom used, and then used only by experts? What's wrong with a culture obtained from studying mathematics that is really used by real people in real life? Is it degrading for a mathematical principle to have a real life application?

"What's the use of this algebra?" the students will ask in an algebra class. "It's a tool subject," they are told, "you will need it when you come to calculus." "What do you use your calculus for?" "Oh, you'll need that for differential equations and for celestial mechanics."

And so the story goes. They never bother to tell you that the way you learned your mathematics isn't the way you're going to use it in physical and chemical research or in engineering. In fact, most engineers will brag that they have never found a use for their calculus since the day they left college. It must have been the wrong kind of calculus, don't you think? Maybe they ought to find a man or group of men that know both engineering *and* mathematics that can write textbooks that will not be out of this world, that will take real problems from real life and solve them the way they are really solved by good engineers.

At the University of Denver, I have been attempting to present what I call a really cultural course in mathematics. I have a year's series designed as a terminal course for those in the college of arts and sciences that are planning *not* to go into technical fields.

In the first two quarters the course is called MATHEMATICS FOR MODERN TIMES, a title I found in the catalogue of Antioch College. Sub-titles for these two courses could be CONSUMER MATHEMATICS, and VOCATIONAL MATHEMATICS. The third course is called PRACTICAL COMPUTATION; in it, slide rule and logarithms and their applications are well covered and, besides, the students do fifty jobs on four types of calculating machines: full-keyboard adding machine, ten-key Sundstrand, Comptometer, and the Marchant or Monroe type of complete automatic calculator. This latter course is also recommended for engineers as an elective.

The general approach in these courses is perhaps not logical, but rather psychological. *Problems are presented as they occur in life and then the mathematics is found to solve them.*

For instance, in the field of financial mathematics, we encounter the problem of finding the amount of $2750 invested for 20 years at 5% interest, compounded semi-annually, or

$$A = \$2{,}750\ (1+0.025)^{40}$$

or

$$A = \$2{,}750\ (1.025)^{40}$$

Obviously, it is impossible to multiply \$2,750 by 1.025 forty times by ordinary methods. We have a magnificent excuse for going into the field of logarithms to work out our problem. After we have done this, we can show how problems like this are most easily solved by interest tables. But it was good culture for the students to find out how those tables were constructed.

As another instance, we suggest that we would like to lay out a circular highway curve. All of you know, of course, that most highway curves are circular, because you have noticed that you need to turn your steering wheel just once at the beginning of the curve and it is adjusted for the rest of the curve, barring ruts. Wouldn't you like to know how the surveyor lays them out, especially when a cliff or a forest would prevent him from taking a tape and drawing a big circle on the ground?

If you are unacquainted with this problem, you do not realize how much good Euclidean geometry is involved in it. This problem alone is a good validation for learning much elementary geometry and trigonometry. If I had more space, I would like to give a more complete explanation of the amount of important geometry involved.

You should see, too, how much analytic geometry is required to work out the area of an irregular field by the coordinate method and how fascinating it is to learn analytic geometry that way. The application of calculating machines to this problem gives an idea what miracles can be performed in this manner. It is disturbing to see a calculating machine on every desk of the local iron works and the Reclamation Bureau manned by men who often do not know and will not learn the full possibilities of these machines. Not that they would learn it all in my course, but there ought to be a good complete course of this type somewhere.

The problem of determining the equal monthly payments necessary to pay off a mortgage on a car or a home involves some high-powered work in geometrical progressions and logarithms that would give even a college algebra student pause, but many students in my classes work out these problems with far less background and far more enthusiasm than most college algebra students. For they are approaching it with the idea that they have a practical problem to solve and they are making good use of some powerful mathematics for solving something that is the basis for several million FHA loans. That kind of thrilling interest you very seldom find in a class where you have the out-of-this-world approach, except, of course, among the abstract geniuses.

These courses are my partial solution to meet what I think is a serious problem in mathematics. It would be much better to have such a course as this written by experts in each field, and a good mathematics teacher to add the drill exercises that would be necessary for a full understanding of the problems involved. Otherwise, the treatment of such problems cannot help but be perfunctory and disconnected.

I could take more space to indicate what means I should employ to keep mathematics in the field of reality and common-sense instead of in the land of make-believe, where it unfortunately is for most dazed students that pass through it. But I think I have given you idea enough of my answer to the question: "IS MATHEMATICS OUT OF THIS WORLD?"

RENEWALS AND NEW SUBSCRIPTIONS

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The Relative Effectiveness of Two Methods of Teaching Certain Topics in Ninth Grade Algebra*

By R. E. Michael

Ball State Teachers College, Muncie, Indiana

The Problem

Two distinct methods of teaching positive and negative numbers, the fundamental operations with them, and the solution of simple equations were subjected to controlled classroom experimentation in this research to determine their relative effectiveness.

One method, designated as Method A, emphasized the use of exercises in thinking, with the exercises built around familiar situations involving time, money, directions, temperature, and others of the type commonly used in textbooks in algebra. Through the use of these exercises, the pupil was expected to discover and understand the fundamental principles and relationships to be learned. The use of numerous practice exercises to bring about efficiency in the operations was supposed to follow the discovery and understanding brought about inductively by the learning exercises. While pupils undoubtedly came to generalize for themselves individually at various times during the experimental period, no statement of the rules of operation was made by teachers or pupils in teaching, reteaching, or pupil discussion.

The other method, designated as Method B, emphasized the use of authoritative statements of the rules of operation combined with extensive practice or drill. No attempt was made, before practice with the respective processes was begun, to explain why the rules operated to give the correct results. Through the process of working with the rules in many exercises the pupil was expected to gain operative efficiency and to acquire understanding of the principles and relationships in the area under consideration.

The descriptions of the essential nature of the two methods given above indicate that Method A was inductive and that Method B was deductive. With this essential difference in the logical plan of the two methods in mind, the reader will recognize that they are related to two distinct theories of learning. Method A bore a close relationship to the organization or generalization theory of learning; Method B, on the other hand was similar to the connectionist or bond theory of learning.

The Nature of the Experiment

It was necessary to enlist the cooperation of teachers in regular public high schools. In order to obtain this cooperation it was obvious that classes would have to be used as found already organized in the schools without any shifting in an effort to match groups. These considerations led to the decision to employ the methods of covariance analysis and resulted in changing certain features of the experimental set-up.

One requirement imposed by the method of statistical analysis decided upon was an initial measure in each of the variables under study. Computation, generalization, and attitudes were considered the basic areas to be examined in relation to the methods, and appropriate tests accordingly were constructed. The instruments designed to measure computation and generalization were each 100 items in length, while the attitude test, actually two tests, had an over-all length of 20 items. The same tests were administered as pretests at the opening of the experimental

* Abstract of a thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Education in the School of Education, Indiana University, October, 1947. Dr. Carl G. F. Franzen, Director of Thesis.

period and again as post-tests at the close. The lapse of time between computation tests was approximately 45 class periods; for generalization and attitudes, it was less and varied considerably among the classes.

The procurement of classes and cooperating teachers was conditioned by the manner in which the materials for use in the experimental teaching were planned. Uniformity in text material used by the classes preceding the experiment as well as uniformity in the order of presentation of topics during the experiment was desired. The material in Schorling, Clark, and Smith's *First Year Algebra*, one of three textbooks used in Indiana at this time, was considered adaptable under the plan. Accordingly, a manual was prepared for the teachers, outlining the methods and giving specific instructions for teaching each section under each method. Certain omissions from the text material were prescribed under each method, and suggestions for handling the topics in keeping with the principles of the respective methods were given. Thus, the selection of the basic textbook restricted the selection of classes to those schools using that book.

Fifteen classes under as many teachers provided the data for the first set of analyses of computation, generalization, and attitudes. The second set of analyses was based upon groups at three levels of ability, sorted without regard for class membership.

In addition to scores on the tests mentioned above, data on socio-economic background were obtained from the use of a questionnaire prepared for this study. The intermediate level of the California Short Form Test of Mental Maturity provided the intelligence test data used in comparing the groups and in the study of the differential effect of the methods at three ability levels which comprised the second set of analyses. Other data recorded for each pupil included grade level, age, sex, and previous instruction in ninth grade mathematics. The manner in which the data were treated will be described in the next section.

After total scores had been established for the pre- and post-administrations of the tests, they were resolved into part scores to provide a more detailed analysis of the effect of the two methods. The abilities measured by these parts or subtests are listed in the analyses to be summarized later.

Some notion of the meaning of generalization in this study as well as examples of the subdivision of the tests into parts may be obtained from the sample test items given below.

From Part 3, "Using signed numbers in writing generalized expressions of oppositeness," the following are offered:

"If dm represents a number of miles south, what would represent twice that many miles north?"

"If we represent 9:00 o'clock A.M. as $+h$ hours before noon, what will represent 3:00 o'clock P.M.?"

"From his base camp a mountain climber goes up j feet to the top of the ridge, and descends k feet on the other side. Let upward be regarded as the positive direction and the altitude of the base camp as the zero point. What is his altitude if $j=k$?"

Further examples have been selected from Part 4, "Determining the sign of the result of a process with generalized numbers."

"Given $p=uw$, write the value of p when $w=-k$.'

"Let any number of miles east be defined as positive and any number of miles north as also positive. Express with positive or negative terms the following: Express f miles east, e miles north, p miles south, and q miles west. What sign will be proper for the east-west direction from the starting point if f is greater than q and p is greater than e?"

"If $tx+py=g$, write the value of g if $t=-p$ and $x=-2y$."

A final illustration is drawn from Part 6, "Interpreting functional relationships in generalized number expressions." It

was in the three parts from which these samples are being drawn that significant differences, described later, were found.

"Does the quantity p/q increase or decrease as p increases?"

"Is it true that when the side of a square is doubled, the area is doubled?"

"Does the quantity $r+s/t$ increase or decrease as t increases?"

The reliability of the computation and generalization tests was computed from the correlation of odd and even scores for stratified random samples of approximately 100 cases each, drawn from pre- and post-test results on the two tests. Reliability coefficients were obtained by applying the Spearman-Brown prophecy formula to the correlations of the odd-even scores. Reliability coefficients for both tests at both administrations ranged from .94 to .96. The reliability of the Attitudes Test was estimated from a test-retest situation with approximately 60 subjects; the correlation of the two sets of scores was .91. No computations of reliability coefficients for part scores was attempted for any of the tests.

The Treatment of the Data

Since the computations involved in the methods of analysis of covariance are carried on from the sums of scores, sums of squares of scores, and sums of cross products, the data were coded and punched on cards to provide for machine tabulation and computation. The high degree of flexibility provided under this arrangement allowed the construction of the frequency distribution tables for socio-economic data, intelligence test scores, and both pre- and post-test scores in the areas of computation, generalization, and attitudes. It was also possible to use any combination of the results needed and to obtain accurate summations for computation purposes.

Supplementing the analysis of covariance, tables were prepared showing the means, standard deviations, and gains between pre- and post-testings for all total and part scores. Furthermore, these figures were displayed for each class and method group. In addition to these computations, the coefficients of correlation between the pre-test and post-test scores for each class were given in these tables.

The general nature of the treatment of the data has been summarized above. The implications of the methods of analysis used will be further apparent in the summary of the findings comprising the next section.

Findings from the Analyses

Since the statistical methods of analysis of covariance provide for the adjustment of final measures in terms of the regression of final upon initial scores, attention in the study of results is focused upon differences in gains between the methods groups. The results of analyses of the scores are summarized in the tables that follow.

It would seem from these data that the two methods may be expected to produce

Table I

The Analysis of Computation Test Results

Part	Difference in Gains	Method Favored	F (Snedecor's)
1. Addition	0.7	A	1.99
2. Subtraction	0.7	A	1.92
3. Multiplication	0.7	A	4.17
4. Division	0.1	B	1.06
5. Evaluation of an expression	0.2	B	0.19
6. Solution of equation	0.7	B	0.71
Total scores	1.2	A	1.23

$F(5\%) = 3.86$
$F(1\%) = 6.70$

TABLE II

The Analysis of Generalization Test Results

Part	Difference in Gains	Method Favored	F (Snedecor's)
1. Substitution of equivalents in general number expressions	0.0	—	0.00
2. Interpretation of + and − numbers and zero in general form	0.1	B	0.00
3. Using signed numbers in writing generalized expressions of oppositeness	0.9	B	10.16
4. Determining the sign of the result of a process with general number expressions	1.0	B	6.19
5. Writing a general expression of number relationships	0.3	A	0.75
6. Interpreting functional relationship in general number expressions	0.5	B	4.44
7. Describing next steps in solution of equations	0.6	A	16.99
Total scores	1.9	B	5.04

$F(5\%) = 3.86$
$F(1\%) = 6.70$

TABLE III

The Analysis of Attitudes Test Results

Part	Difference in Gains	Method Favored	F (Snedecor's)
1. Attitude toward mathematics in general	0.3	B	2.10
2. Attitude toward algebra	0.1	A	15.20

$F(5\%) = 3.86$
$F(1\%) = 6.70$

approximately equivalent gains in the area of computation with the possible exception of the process of multiplication. In this area, the inductive procedures of Method A seemed to have produced significantly greater gains. In other respects the tendency of the data is to favor Method A within the limits of possible chance fluctuations.

The conclusion reached in the light of the results of this study regarding generalization was that the deductive procedures of Method B produced significantly greater gains than the inductive procedures of Method A.

It was concluded that the differences in instructional method produced no significant changes in the attitude of the pupils toward mathematics in general. Method A however, produced better results in attitude favorable to algebra, the subject of immediate interest.

DIFFERENTIAL EFFECT OF METHODS AT DIFFERENT ABILITY LEVELS

Three equal levels of ability were established on the basis of total intelligence test scores, and the groups obtained by sorting the punch cards for each method on this basis were studied. These analyses were limited to computation and generalization test scores.

The technique of analysis of covariance involving two categories with disproportionate sub-class frequencies was applied to the total scores for each test. No effort was made to study the part scores in relation to ability levels.

According to the means derived at each level for each method group, the deductive method produced slightly better results in total computation scores than the inductive method at the high and medium levels of ability. On the other hand, a slight difference favoring Method A was

observed at the low level. The test of significance showed, however, that these observed differences between methods at the different levels could be due to chance fluctuations.

While the test of significance failed to reveal differences in gains in generalization between methods that could not be attributed to chance, observation of the gains suggested that the high level under Method B gained more than the medium ability level. This observation suggests interaction between levels and methods favoring Method B at the high level of ability.

The studies of interaction here reported are suggestive of hypotheses which would bear investigation. However, much more evidence would have to be presented before these tendencies could be supported with confidence.

General Conclusions

There was a tendency for the over-all results of the study to favor the deductive procedures of Method B. The results from the Computation Test did not show significant differences with the exception of the process of multiplication, in which instance Method A produced significantly higher scores. The results from the Generalization Test were distinctly favorable to Method B, as shown by the significant differences in gains observed in total scores and in three areas associated with the part scores. While it will be recalled that significant differences were observed in intelligence that would favor the group instructed under Method B, the results of the analyses of covariance, which make allowance for initial status in the variables under study, would seem to constitute a strong argument to offset differences in intelligence test scores.

In the light of the above statement, the outcome of the study seemed to constitute somewhat of a challenge to the contentions of modern educational theorists for inductive teaching procedures, as far as the teaching of the topics of positive and negative numbers is concerned.

It can be said with safety that the results of the study are such as to throw doubt upon the advisability of making broad assertions regarding the best methods of teaching topics in high school algebra unless a body of supporting evidence from classroom experimentation has been accumulated.

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The Mathematical Behavior of Minerals

By THEODORE C. RIDOUT

Editorial Staff of Ginn and Company, Boston, Massachusetts; formerly master at Browne and Nichols School, Cambridge, Massachusetts

THE purpose of these notes is to show the possibility of making greater use of crystal and cleavage forms of minerals in connection with courses in mathematics. I first became interested in the subject when teaching plane geometry, and over a period of years made considerable use of mineral forms to illustrate geometrical principles. My students found this material interesting and often quite astonishing, as indeed it is. I am convinced that it can be more widely used, in both classroom and textbook, with benefit.

The most spectacular mineral for illustrating mathematical cleavage is calcite, or calcium carbonate in naturally crystalline form. Specimens of cleavable calcite can be obtained cheaply from Ward's Natural Science Establishment in Rochester, N. Y., or any mineral supply house. A sample somewhat smaller than one's fist is shown in class, and is noticed to have flat surfaces which are perfect parallelograms. When struck a quick blow with a hammer the specimen will break into a number of pieces, all close copies of the original specimen.

The angles between these cleavage faces are constant, and can be accurately determined with a reflecting goniometer. Suffice it for our purposes to point out that the acute angle which appears in these forms is approximately 74° 55′. Since opposite surfaces are parallel, the supplementary angle is of course 105° 5′. No matter how microscopically small the fragment one chooses to examine, these angles are in evidence, and are, in fact, constant.

Halite, or common salt, breaks at an angle of exactly 90°, and ordinary table salt, if examined closely, may be seen to be composed of perfect cubes. The mineral galena, or lead sulfide, also has this perfect cubic cleavage. Orthoclase (Greek for "straight break,") shows perfect rectangular cleavage in two planes only, so that it is common to find specimens of rectangular cross section, but unevenly broken across the ends. Microcline ("small angle") is chemically identical with orthoclase, but has a somewhat different molecular arrangement which causes it to break at an angle of 89° instead of 90°. Augite and hornblende are also much alike; but augite has cleavage angles of 87° and 93°, the other, 54° and 126°.

Not only cleavage forms but also the crystal habit, or natural arrangement of crystal faces, shows similar mathematical properties. Galena crystals, for example, occur in cubes, octahedrons, and other regular forms. Fluorite does the same. Quartz crystals grow in handsome hexagonal prisms, each terminated by a pyramid. The cross section is always a perfectly equiangular hexagon, but is not necessarily equilateral.

The most remarkable cleavage form is that of sphalerite, which is the principal ore of zinc. Material of good quality can be broken along its lines of natural cleavage to give a fine rhombic dodecahedron, or twelve-sided figure, whose faces are perfect parallelograms. Even ordinarily good material can be made to reveal half a dozen such faces with a bit of careful chipping. The regular pentagonal dodecahedron is not found in minerals, since it is crystallographically impossible.

Crystals of garnet often grow in perfect rhombic dodecahedrons, sometimes with neatly beveled edges which are due to the superimposition of the icosahedron. Almandite garnet from Alaska yields fine dodecahedrons the size of a plum. Andradite sometimes grows in clusters of small crystals of the same form. Other types of

garnet show a 24-sided figure known as a trapezohedron, whose faces are irregular but congruent quadrilaterals.

The reason for these phenomena can be demonstrated with a set of balls or marbles and a suitable frame to hold them together. For instance, a set of fifteen pool balls can be most compactly assembled by means of a triangular wooden frame, which is commonly used in playing the game of pool. The balls fit more compactly in this form than they would in a square, or in any other shape.

Now if you will pile more pool balls upon the fifteen in their frame, you will finally have a pyramid which is a perfect regular tetrahedron, or four-sided triangular solid.

Molecules follow a similar pattern, for the same reason. They fit themselves together as compactly as their individual forms will permit, forming a three-dimensional solid. Fluorite molecules behave exactly like so many pool balls. Extend the figure indefinitely in space, and you have a good picture of the molecular arrangement of any crystalline substance.

Return to our pile of pool balls. Suppose we remove the top ball and the corner balls from our tetrahedron. We are heading directly toward an octahedron. On a larger model we can easily obtain a perfect regular octahedron by omitting from each vertex the proper number of layers. In the same way we can shave down the corners of a cube until we get an octahedron. By extending alternate faces of the regular octahedron we can restore the tetrahedron. Thus these forms are mathematically interrelated, and a mineral like fluorite may show more than one of the forms, because they are all based on the same space-lattice.

Different molecules arrange themselves in different patterns, much as if we should use blocks of different shapes in building walls. The form of the finished product is determined by the nature of the basic unit.

It is evident that such material is interesting and useful in connection with both plane and solid geometry. Some student is always asking "What is the use of all this geometry?" Here is a convincing answer.

As a matter of fact, the discovery of cleavage in calcite by Wollaston, in 1812, was the beginning of the scientific study of minerals. Later the law of the constancy of interfacial angles was established. These angles have been precisely measured, and recorded in textbooks such as Dana-Ford, *Textbook of Mineralogy*, where lists of interfacial angles are given for each mineral species. Such figures are frequently useful in distinguishing between minerals, and are the backbone of crystallography.

For excellent photographic illustrations, see George L. English, *Getting Acquainted with Minerals*, McGraw-Hill, 1936. A list of the most appropriate ones in the book is attached. A glance through any text in mineralogy will show the highly mathematical nature of the subject. The mathematics, however, is of *nature's own making*. Minerals form in the earth according to mathematical necessity, not to please the professors. From this fact arise philosophical implications which are also important.

Finally, minerals are fascinating and beautiful in their own right, and make an appeal to all. No school should be without a collection of suitable specimens; no textbook in elementary mathematics should come from the press without some mention of the mathematics of minerals.

The following illustrations are to be found in George L. English, *Getting Acquainted with Minerals*, McGraw-Hill, 1936.

page	
66	striated tourmaline
67	striated quartz
68	selenite xls
69	attached quartz xls
70	amethyst xls in parallel position
71	reticulated grouping of cerussite xls
*73	diagrams of staurolite, 60° and 90° twins
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*77	cubical cleavage of halite (salt)
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152	fluorite, group of cubical twins
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158	diamond, triangular growth projections on octahedron
*184	galena, xls on dolomite
188	dolomite, saddle-shaped group
203	rutile, repeated twinning
211	franklinite xl
*227	garnet xls in mica schist
229	muscovite xl (hexagonal)
*235	smoky quartz
248	analcite xl on rock (trapezohedron)

* = best pictures
xl = crystal

Minerals can be obtained from Ward's Natural Science Establishment, Box 24, Beechwood Station, Rochester 9, N. Y.

Another Approach to (−a) (−b)

By M. Van Waynen
Berkeley High School, Berkeley, California

Now that V. Sites[1] has continued the discussion on the topic of the logical approach to $(-a)(-b)$ started by H. Van Engen's[2] article it seems only fair that the rest of us forward our pet methods. The method I use has been very successful because it appeals to the common sense and appears simpler.

The explanation that follows was first brought to my attention by F. L. Griffin, Professor of Mathematics, Reed College, Portland, Oregon, in a talk given by him at the University of Southern California several years ago. The method is different, logical, and should merit the attention of all mathematics teachers who still use the rather stupid definitional approach to $(-a)(-b)=(+ab)$.

We again assume that the class is acquainted with negatives and what they mean. Negative numbers have been added and subtracted and rules for doing so have been formulated.

Now we simply must ask for agreement on

$$(+4)+(-4)=0$$

and

$$(+3)=(+3)$$

Invoking the multiplication axiom gives us

$$(+3)(+4)+(+3)(-4)=0$$

We know that

$$(+3)(+4)=(+12)$$

What does common sense dictate that $(+3)(-4)$ equals?

Using the subtraction axiom we get

$$(+12)\doteq(+3)(-4)$$

Knowing that the product of a negative and positive number is negative we can go a step further and say

$$(+4)+(-4)=0$$
$$(-3)=(-3)$$

the product of which is

$$(-3)(+4)+(-3)(-4)=0$$

Since

$$(-3)(+4)=-12$$

we get

$$(-12)+(-3)(-4)=0$$

Using the subtraction axiom again gives

$$(-12)=-(-3)(-4)$$

or,

$$(-3)(-4)=(+12)$$

This method does not seem as involved to me as the others presented and can be understood by even the poorer pupils. The advantage of this system over the others lies in the fact that it puts the whole problem on a common sense basis. The product of two negatives has to be a positive for that is the only way in which sense can be made!

[1] The Mathematics Teacher, December 1947. p. 384.

[2] The Mathematics Teacher, April 1947. p. 182.

A Comparison of Traditional Teaching and Personalized Teaching in Ninth Grade Algebra

By MARGARET SCHULTZ AND M. M. OHLSEN
The State College of Washington, Pullman, Washington

IT IS an accepted fact that the whole individual functions as he learns. The pupil's feelings as well as his intellectual analysis of the mathematical problems come into play in the learning process. Emotional blocks, feelings of insecurity, conflict at home, and all other personal problems affect the pupil's ability to perform in the classroom. Evidently there is something in the learning process that penetrates deeper than mere memorization of facts.

On the other hand, as teachers, we see the value of the mathematics that we teach yet we often fail to make pupils see its usefulness. What the pupil learns takes on a new meaning when he can tie it into his daily problems and his own personal goals. Therefore, it becomes a matter of personalized teaching and a matter of accepting in practice, as well as in theory, the fact that learning is an individual matter. Certainly, this concept isn't new even though it isn't always practiced.

It is logical that you might ask, "Why bring this matter to our attention now?" It has been raised here for two specific reasons. First, it has been shown by Casner and Nyberg,[1] Orleans, Saxe, and Cassidy,[2] and Ohlsen[3] that high school students have not mastered the basic mathematics that the ordinary citizen should know. Second, it has been said that individual remedial assistance helps correct this difficulty among high school students. Even though this second argument has been accepted in theory, it has been rejected by many high school teachers as an unusable solution because classes are so large. We realize that the typical classroom teacher will not be able to give special remedial help to every individual member of the class. Furthermore, we recognize that learning goes on through exchange of help among students even though the teacher can not make the rounds to each individual pupil.

Moreover, we contend that a planning committee should define basic mathematics for their particular community and then set up evaluation procedures to determine growth periodically. Since fall testing gives a measure of "take home pay" in knowledge retained, depreciated by forgetting through the summer, we recommend fall testing. Testing should be followed-up by a careful analysis of test results. The classroom teacher should build a card file which describes individual strengths and weaknesses as she studies the tests. In fact, the test results should be studied with the pupil in an individual conference to enable him to help select the types of problems for which remedial instruction will be given. It is important that the pupil recognize the need for remedial work and that he feel that he has a reasonable chance of correcting these deficiencies during the school term. As these remedial teaching cards are prepared they should be alphabetized by classes so that they will be readily available for remedial teaching. The availability of

[1] A. Casner, and J. Nyberg, "What Do High School Graduates Know About Arithmetic?", *Journal of Business Education* 15: 17–18, September, 1939.

[2] J. Orleans, E. Saxe, and W. Cassidy, *An Analysis of the Arithmetic Knowledge of High School Pupils with Emphasis on Commercial Arithmetic*, School of Education, College of City of New York, 1943.

[3] M. Ohlsen, "Control of Fundamental Mathematical Skills and Concepts by High School Students," THE MATHEMATICS TEACHER 39: 365–371, December, 1946.

diagnostic information encourages the teacher to use these facts as well as to do more remedial teaching. What is more, she can increase the number of individual teaching experiences for pupils by using her better pupils as assistants.

This particular study was set up to compare the relative merits of teaching first-year algebra through the use of traditional group methods with a plan which emphasizes individual diagnosis and personalized teaching. We compare the growth between the two groups in both basic mathematics and first-year algebra.

This report actually reviews two independent studies. At the beginning of each experiment, both the experimental groups and the control groups were given the *Cooperative Algebra Test, Elementary Algebra Through Quadratics,* Revised Series Form S. The test was also given at the close of each study. To obtain the measure of growth in every-day mathematics, parallel forms of *Ohlsen's Test of Mathematics Essential for the Ordinary Citizen* were used. The criterion used to determine growth in each area was the difference between initial and final scores.

Since it wasn't possible to get matched groups in the experimental and control groups it was necessary to teach each group by prescribed procedure and then to use the data from only the matched pairs in the two groups. On the average there were about twenty-five students in each class. Individual students in the experimental and control classes were matched on the basis of "work attitudes," "achievement in arithmetic," "mental test scores," and "sex." The same instructor taught all four classes.

The first experiment was begun in November, 1946. The classroom working relationship was already established. Hence, the experimental group resisted a change in procedure.

To insure difference in instruction, five guides were developed to govern activity in the experimental group. These will be presented together with the corresponding activity for the control group.

The experiment was carried on for 125 days, eight of which were used for unit tests, semester examinations, and the testing for this study. During this time the instructor kept a diary of class procedure with the approximate length of time spent on the various phases of instruction. This diary shows that during the 117 teaching days of the experiment, an average of 7.9

Teaching Approach for the Experimental Group	*Teaching Approach for the Control Group*
1. Routine jobs such as taking roll and checking papers were done as efficiently as possible. Five minutes were allowed for these tasks.	1. Same as (1) of the experimental group.
2. Discussion of theory and application was reduced to a minimum. The plan was to spend not more than fifteen to twenty minutes on this phase of the work.	2. Most of the class period, thirty to thirty-five minutes, was devoted to the presentation of theory and application, and to group remedial work.
3. At least half of the period, twenty-five minutes, was reserved for individual instructions. A few of the better students were asked to assist the teacher during this part of the class period.	3. About ten to fifteen minutes were used for a study period. During this time, students could get individual help but they were not urged to do so.
4. After the diagnostic test in basic mathematics had been scored, the student and teacher together analyzed the student's errors. For each student crucial weaknesses were listed on a card. This information was the basis for remedial work.	4. The teacher made no particular effort to diagnose student difficulties.
5. The application of algebra to life situations, meaningful to the student, was stressed throughout the course. Students were urged to discover these applications and to write their own problems.	5. The work was based primarily on textbook material.

minutes per class period was used for routine tasks in the experimental group; 8.9 minutes in the control group. The plan called for only five minutes. However, if a record has not been kept, possibly more time would have been consumed in both groups. Keeping a diary tended to point out such time-consuming activities as taking roll, checking assigned work, and making announcements.

Summaries of the time spent on theory, application, and group remedial work show an average of 16.6 minutes per class period for the experimental group and 23.4 minutes for the control group. At times the experimental class asked so many questions, seemingly of interest to the entire group, that it was difficult to curtail group discussion.

The third guide governing instruction in the experimental group called for student assistants. During the first semester, two students who had completed only one semester of algebra the previous year were reviewing that work by auditing the class every other day. While these two students were happy to act as assistants, other able students were not interested. In a short time, however, these same students were really acting as assistants without formal recognition, for pupils were encouraged to help one another. The class gradually grouped itself. Most of the groups now had a "specialist." Occasionally even a relatively weak student had the satisfaction of giving help.

The answer sheet for Ohlsen's test provided "working space" for each problem. A study of this part of the test was the basis for the diagnosis in the fourth teaching guide. Every student listed all the types of errors he had made. Now he was ready to select the kind of problem which troubled him most frequently; then, the one of second greatest difficulty. Thereafter, the student, using the same procedure, listed the remaining areas of difficulty.

In helping the student prepare his diagnostic card, the teacher kept two questions in mind: What can the student learn? What will he use? Accordingly, some of the less capable pupils were encouraged to list just a few of their most pressing problems. On the other hand, every error of the more capable student was noted for remedial emphasis.

In order to keep individual goals in sight, the diagnostic cards were filed and placed where both students and teacher could refer to them easily during the class hour. As soon as any item on the card was mastered, it was crossed out. Other weaknesses were added whenever they appeared.

The fifth teaching guide for the experimental group stressed the application of algebra to life situations. This process encouraged students to discover these applications and to write their own problems. Students showed little interest in writing problems, but they were more enthusiastic in reporting problems and mathematical items which they found in their reading and in other contacts.

Table I compares the *gains* made by both groups in both algebra and basic mathematics. "B" and "G" refers to matched pairs of either boys or girls.

Even though the differences were not considered to be statistically significant it is interesting to note that there were positive differences in favor of the experimental group. In addition to the tested differences the following personal reactions stood out:

1. Pupils from the experimental group felt that they had had more "how" and "why" learning experiences.
2. Practically every pupil from the experimental group expressed appreciation for the amount of individual help.
3. From the control group came many requests for individual help.

In the second experiment the control group was taught just as it was in the first experiment. The following differences were incorporated into the second experiment:

TABLE I

A Comparison of Gains Made in Experiment I

Matched Pairs	Gains in Algebra			Gains in Basic Mathematics		
	Control	Experiment	Difference	Control	Experiment	Difference
B-1	24	31	7	5	8	3
G-1	16	23	7	−1	2	3
G-2	22	30	8	−3	−2	1
G-3	18	24	6	6	3	−3
G-4	12	32	20	5	5	0
B-2	18	35	17	3	1	−2
B-3	20	19	−1	7	8	1
G-5	18	14	−4	−4	−1	3
G-6	30	22	−8	0	7	7
G-7	14	9	−5	5	7	2
B-4	22	16	−6	2	3	1
Mean Gain	19.5	23.2	3.7	2.3	3.7	1.5

N.B. Data is expressed in raw score gains.

1. The study began the first day the classes met.
2. The second experiment was concluded at the close of the first semester. Hence, it consumed approximately fifty per cent less time than the first experiment.
3. Student assistants were *selected by the class* to help with instruction. Before electing such students, the class talked about the criteria for a wise choice. It was decided that assistants must know how to do the work and be able to explain it to someone else. They must also be dependable, friendly, and patient.
4. Student helpers were not required to hand in the minimum assignments. In the first place, no student could be expected to devote most of the class period to helping someone else and then cheerfully do all his own work outside of class. Secondly, by the time the student has worked with other students for the greater part of the period, he would probably be familiar with most of the required work. The class had been told that they were expected to turn in all assigned work, but that their grades would be based almost entirely upon short tests and unit examinations. One or more of these short tests were given every week. Inasmuch as student helpers participated in the entire testing program, they realized that their obligation was not decreased by their status in the class.
5. Whenever the class had no questions, the student helpers worked on more difficult problems. This time was also utilized by the teacher to instruct the assistants on how to help students.
6. Group checking of assigned work was eliminated entirely. Enough answer books were provided for textbook assignments so that each student could check his work as soon as he had completed it. The student helpers recorded completed work after the pupil had found and corrected his own mistakes with the help of assistants if help were needed.
7. Time spent in introducing new material was shortened. The class often began work without any help other than studying the examples and explanatory material in the text. On the other hand, when the majority of the group was unable to operate alone and the assistants and teacher together could not keep up with the requests for help, more group instruction was given. A summary of time spent on the various phases of

TABLE II

A Comparison of Gains Made in Experiment II

Matched Pairs	Cooperative Algebra Test			Ohlsen Test		
	Control	Experiment	Difference	Control	Experiment	Difference
B-1	17	19	2	9	11	2
B-2	15	15	0	0	7	7
G-1	22	25	3	11	17	6
B-3	2	14	12	8	21	13
G-2	14	17	3	1	7	6
B-4	12	15	3	−1	6	7
G-3	20	15	−5	13	12	−1
G-4	18	13	−5	1	5	4
Mean Gain	15.0	16.6	1.6	5.3	10.8	5.5

instruction reveals that group instruction averaged 12.3 minutes per day while individual instruction consumed 30.1 minutes per day. The remaining portion of the fifty minute period was used for short tests and group planning. For the control group, 30.6 minutes per day was devoted to group activities; 14.2 minutes, to individual study. This summary is based on the diary of class procedure kept by the instructor.

8. A greater effort was made to interest the class in writing their own problems. More class time was devoted to learning how to write problems.

The first experiment was approximately one and one-half times as long as the second experiment. It would seem reasonable, then, to expect that the mean difference between gains should be larger for the first experiment. In the case of the algebra test, this was true. Mean difference between gains was 3.7 for the first experiment and 1.6 for the second, or a little more than twice as great in the first experiment as in the second. In the Ohlsen test, however, mean difference between gains was nearly four times as great as in the first experiment, the exact gains being 1.5 and 5.5.

Several factors may have a bearing on the increased gains for the Ohlsen test.

1. The teaching procedure was improved in the second experiment.
2. The teacher had had almost a year's experience in applying the technique prescribed for the experimental class. This point might suggest that teachers should not be unduly discouraged if the first attempt to use an individualized technique does not produce marked improvement in results.
3. The second experiment was started on the first day that classes met; whereas about two and one-half months of school had passed before the earlier experiment was organized. Possibly the second study capitalized on the enthusiasm with which students usually begin the year. In any event, pupils in the second experimental class showed more interest than those of the first in working on weaknesses listed on their diagnostic cards. This greater interest in remedial work resulted in less class time being spent on algebra and may in part account for slightly smaller gains in algebra for the second study. Of course, the time spent in the experiment was another factor.

Student evaluation of the teaching methods employed in the two algebra classes gives a partial evaluation of those important personal factors which the tests used in this experiment did not measure. Responses from the experimental class will be discussed first.

The experimental class was definitely a happy group. Two-thirds of them had no suggestions whatsoever for improving the instruction. The student who said, "I

have no suggestions. I am perfectly satisfied with the way things are done," might have spoken for all of these pupils. The suggestions were minor and none of them involved changing the teaching technique.

Only one student failed to mention student helpers as one of the things he liked about the class. This pupil probably had student helpers in mind in saying, "We have all been given enough freedom so that we could talk over anything we did not understand." Many others made similar comments. There was a fine spirit of cooperation. One student remarked, "I liked the way everyone worked together for the good of everyone."

A number of students expressed appreciation for being allowed to work pretty much at their own rate. They enjoyed the added responsibility it placed on them. They also liked to help plan the work and to offer constructive criticism. The teacher also enjoyed the added responsibility assumed by the students. There was less pressure on the teacher; hence, teaching was more pleasant.

There was some evidence of a changed attitude toward mathematics. One student thought that the method used was at least partially responsible for arousing his interest in the subject. He had never liked mathematics before and had intended to take as little as possible. Now he is planning to take three and one-half years of mathematics in high school. Other students revealed an added interest in mathematics in their informal talks with the teacher and through the types of questions they asked.

The evaluation made by the control class was less detailed than that of the experimental class. Many of them appeared to have an indifferent attitude toward the class procedure. One student could think of nothing he liked and had no suggestions for improvement. The majority agreed that there should be more time for individual help and more time for work in class. A fairly large proportion of the group throught assignments were excessively long. No more was expected of this class than of the other algebra class, yet no one in the experimental class felt that too much was expected of him. A few of the experimental group made specific comments to the effect that they had had an ample amount of time to complete their work. Practically all of the control group wanted more individual help.

The experimental class in the second study apparently made a better adjustment to the teaching situation than did the experimental class in the first study. This point suggests that in introducing a new teaching technique, it is important to do so at the very first of the year. Moreover, this experiment also suggests that it is wise to take time to get acquainted and learn how to plan together. In the first experiment the group pattern had been established before the study began. In the second experiment only one teaching method was used. Hence, the students participated more enthusiastically.

CONCERNING THE ORDERING OF YEARBOOKS!

Will the readers of "The Mathematics Teacher" please order the Yearbooks of The National Council of Teachers of Mathematics directly from "The Bureau of Publications," Teachers College, Columbia University, 525 West 120th St., New York 27, N.Y. Do not send orders or money to "The Mathematics Teacher" as this will only delay your receiving the Yearbooks, and will cause us a great deal of trouble and unnecessary expense.—Editor

Families of Hyperbolas in Navigation

By Rowland C. Anderson
State Teachers College, Saint Cloud, Minnesota

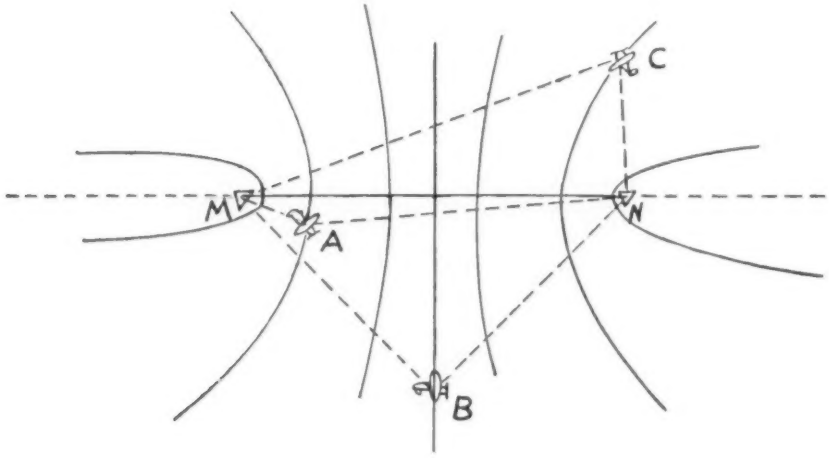
In the darkness 10,000 feet below is the angry Atlantic lashed into a foaming spray by winds of 150 knots or more. Between us and the stars tower five miles of ugly cumulo nimbus clouds lit up at frequent intervals by blinding flashes of lightning. The tropical rain hangs about us like a gray sheet, almost a solid wall of water. This is a hurricane.

We had taken off at dusk and now had been battling the fury of the elements for nearly ten hours. With no friendly stars to guide us, with winds changing from a dead calm at take-off to the present howling gale, with violent currents of air tossing our plane 3,000 feet in the air in a few seconds and then as quickly dropping us perilously near the waves below, with innumerable changes of heading to avoid the worst thunderstorm areas, and with the radio a crackling madhouse, what a night for a navigator! Yet to give Hurricane Central the information it needs to warn a worried coastal area of where the whirling monster will probably strike, to give people time to evacuate their homes and batten down for the big blow, the navigator must report the present position of the storm center and that position has to be good. Radar reports the eye of the storm now bearing 010 degrees—distance 40 miles. The navigator turns to a black box beside him, lights up a screen with two rows of dancing pinpoints of eerie green light. In the field of grass, he picks out two "pips" of light that stand ready. He turns a couple of dials on the face of the instrument; and as the pattern on the scope changes form, he writes down a number. Once again he goes through the routine. Then on a strange chart with a maze of blue, green, purple and brown lines, he finds a pair of numbers corresponding to the numbers he has written down. He draws two lines on the chart, and X marks the spot. He is there and he is sure of it. Radio crackles out a message and the vital information is on its way.

Such is the power of LORAN, the wonder navigation aid. Mathematics? The lines on that navigation chart are families of hyperbolas. Each line is the locus of the points the difference of whose distances from two foci is a constant. At the foci in these systems are radio transmitters approximately 300 to 500 miles apart. About 25 times a second, "master" station M sends out a pulse of radio energy. This pulse travels in all directions at 186,000 miles per second. The pulse is received at "slave" station N and triggers a similar pulse to be transmitted from N. In a plane or ship at sea, these signals are picked up on a special pretuned radio receiver. The signals are relayed to the Loran indicator, an electronic device for measuring the time interval between the receipt of the two signals from M and N in microseconds (millionths of a second). This time difference is the basis of the hyperbolas on the Loran chart.

For illustration, in the following diagram we will take stations M and N so located that the time required for the radio pulse to travel from M to N is 2000 microseconds. A plane at A would receive the pulse from M in 500 ms. For the same pulse to travel from M to N and then back to A would require 3600 ms. At A, the time difference shown on the Loran indicator would be 3100 ms. At any point on the perpendicular bisector of MN such as B, the time for the signal to travel M-B or N-B would be the same; so the time difference at B would be 2000, the time for the pulse to travel M-N. At C, if it takes 2100 ms. for the signal to travel M-C, and 3000 ms. to travel M-N-C, the interval between signals would read 900 ms.

Any time difference shown by the in-



dicator would enable the navigator to place himself definitely on one and only one of the hyperbolic lines on the chart. By taking a similar reading on a second pair of stations, he could get a second locus. The intersection of the two lines gives him a fix. The position can be verified by taking a reading on a third pair of stations if desired.

The time required to make the necessary readings and plot a two or three line of position fix varies from 2 to 5 minutes, depending on the skill of the operator and the conditions of operation. The computation involves only counting and simple addition. The maximum error of the position obtained is less than one per cent of the distance from the transmitters, so that 500 miles at sea the position would be accurate within 5 miles. As the plane approaches the coastline between the stations, the accuracy becomes a matter of yards. Loran is practically unaffected by adverse weather except as extreme static makes the reading more difficult. It has an effective range of about 700 miles in day time. At night, this range is doubled through the use of sky waves reflected from the heaviside layer of the ionosphere.

Loran facilities are available in most areas where our planes make long over-water flights. An extension of the systems called SS (sky wave synchronization) is making practical the use of the gear over land as well as over water. Loran is providing a vital safety factor so necessary with the increased speed and range of military and commercial aviation.

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The Blight of Mathematical Slang

By Lurline Stewart
Hinds Junior College, Raymond, Mississippi

As a background for my remarks, let me say that my teaching career includes the teaching of mathematics in a saw mill town where one half the people owned the town and the other half worked for the first half; in an agricultural high school where the majority of the children came from farm homes; in a smaller rural community in perhaps the poorest county in the state; in a town of around 1200 where culture and civic pride are the order of the day; in one of the largest state universities in the south; and finally in one of the best junior colleges in Mississippi. Students in each of these schools use mathematical slang, and I cannot say that I have found it any more prevalent in one place than another.

Perhaps you are a person who has not paid a great deal of attention to what slang is.

Webster says that slang is "language comprising certain widely current but usually ephemeral terms, (especially coined or clipped words, or words in special senses or phrases) having a forced, fantastic or grotesque meaning or exhibiting eccentric or extravagant humor or fancy." Shall we then object to mathematical slang because it is fantastic, or fleeting, (I wish it were!), or because it offends the ears of a cultured person? I would not condemn it on this basis alone. Rather, I would say that mathematical slang becomes a blight or a curse when it interferes with the true throught processes in real learning and thinking.

Thomas Edison once said that after examining ten thousand college graduates, he had come to the conclusion that "thinking is unnatural." The average pupil often avoids everything that requires him to think; and the present trend seems to be for teacher and text to follow the line of least resistance. We flatter ourselves that we are teaching when we show the pupil how and give him many examples to do. We hope this will fix a habit. Such a procedure is so wrong that I venture to say that it is not teaching.

To me, the slang expression "cross-multiply" is the one used most frequently and the one that lends itself to a greater number of misinterpretations than any other. Just what is meant by cross-multiplication? In searching the literature I find the expression, cross-product, used in connection with the product of two binomials. In fact, the rule is stated thus:

The first term in the product is the product of the first terms in the given binomials and the last term in the product is the product of the last terms of the given binomials while the middle term of the product is the sum of the cross products, the cross products being indicated by curved lines drawn above the first and fourth terms and above the second third terms.

However, that is not what my students nor yours have in mind when they say "cross-multiply." The modern trend in algebra is to write a proportion as an equality of two fractions. In clearing this equation of fractions, the student says cross-multiply. Now, if this were the end of the matter, I would not be alarmed. This same student in dealing with the equation $\frac{1}{4}+\frac{1}{6}=\frac{1}{x}$ will assure you that $4\times6=24$, so $x=24$, even though the equation arose from the word problem "If one man can do a piece of work in 4 days and another in 6 days, how many days are required if they work together?" Furthermore, if asked to solve an equation of the type $\frac{7}{x+1}-\frac{8}{x+2}=\frac{5}{x+3}$, we get the astonishing reply that $56(x+3)=5(x+1)(x+2)$. But this is not the end yet. Given a multiplication $\frac{x}{7}$ by $\frac{5}{6}$, the thought-

less student again says $6x=35$ although there is no connection between multiplying two fractions and solving an equation containing fractions. It is true that in the normal order of things multiplication of fractions precedes the solution of equations containing fractions, but a student on hearing the magic phrase "cross-multiply" becomes charmed with its possibilities (and they are vast), immediately discards previously learned ideas of fractions and glibly replies "cross-multiply" when called on to recite. Just what there is in a student's nature that causes him to exhibit such perverseness I do not know. But it seems to me that the frequency with which a student misuses the term is in inverse proportion to his general intelligence. Just last week one of my weaker college algebra students who apparently had been permitted to use the expression assured me that the proper way to expand a second order determinant was to "cross-multiply."

A splendid high school teacher whose students come to me as college students once remarked to me, "When I teach them the best I can and still my slow students will not learn, I just give in and let them say "cross-multiply." My reply to her was that of all students who should never be permitted to use the expression, those are the ones. They are the students who when they reach college exhibit no comprehension of the real meaning of algebraic language.

Perhaps you permit your students to use the expression. May I ask where you found it? Wat it in the high school text which you were taught? Was it in a supplementary text? Did your teacher permit you to use it—or did he use it before you? Did your students first initiate you into the use of the word? In examining current state adopted texts and those recently discarded, I find no mention of "cross-multiply" as an algebraic process. In fact, I have never seen this expression used in any mathematics text in the sense of clearing an equation of fractions.

Another slang expression used in connection with the solution of equations containing fractions is "throw away the denominator." Some students who have been weaned from saying "cross-multiply" are occasionally permitted to "get a common denominator for both sides of the equation, draw one long line, put this denominator under the line, divide each denominator into the common denominator, multiply the numerator by the respective quotients, placing the equality sign in its relative position from the line above." Now, we have an equation as the numerator of a fraction that has a perfectly good denominator. Another mathematical oddity! His next remark is positively astounding. He says, "We've gotten both sides to the same denominator. Now, throw away the denominator." I've often wondered what became of these thrown away step-children. Of course the modern texts give a concise rule—clear and very explicit—for clearing an equation of fractions. This simple little rule which would preclude all the above nonsense is stated: *multiply each member of the equation by the least common multiple of the denominators.* Try using it. You and your students will find it much more satisfactory than the artificial device called "cross-multiply."

Turning now to geometry, I deplore the use of the abbreviations s.a.s. = s.a.s. etc. as the symbolism for the theorem "If two sides and the included angle of one triangle are equal respectively to two sides and the included angle of the other, the triangles are congruent." Mathematics teachers wiser than I advocate the use of these abbreviations and c.p.c.t.e. Of course there is much to be said for the brevity, but I question whether any save the better students really understand what they are saying when they use them. In fact, there is an unfortunate combination of letters which when used innocently or otherwise by some teen-aged boy and spoken aloud in a geometry class as the answer to why the triangles are congruent that always produces hilarity and I think justifiable

mirth. I think that is is almost as bad for him to say "angle, side, angle" as to use the abbreviation a.s.a. I am making a plea for a complete sentence as an answer to a respectable question.

Another expression which we permit the sixth graders to use but which should be abolished by the time the students reach high school is "turn the fraction upside down or over." To *invert* the fraction is a much kinder act than to turn it upside down. If something is to be turned upside down, we get an idea of confusion or of an upset. Now an inverted fraction is still a modest, self-respecting individual and quite all right in its own proper place. We are assuming of course that this inverting is the correct procedure.

If your students wish to express the product of a monomial and polynomial or of 2 polynomials, do they say "x onto $(a-b)$? Webster says that onto means to a position on or against. Well, x onto $(a-b)$ might be thought of as x against $(a-b)$. But that is absurd. Some texts recommend x parenthesis $a-b$, others x times the quantity or expression $a-b$. Usually a person can indicate the proper grouping by the inflection of his voice.

I have found that students employ the term "transpose" very glibly without the faintest idea of the fundamental operations involved. If you permit the use of "transposition," by all means, justify its existence.

Formerly, the word "cancel" was not in as good repute as it is now. It appears in most standard texts as having the meaning, divide. For instance in the rule for reducing fractions, we find "Factor the numerators and denominators into their prime factors, cancelling the common factors." All right, let's cancel the factors, but let's not cancel them out! If they are cancelled, they are *out* so far as we are concerned. This of course does not help the weaker student who is unable to distinguish between cancellation of factors and the addition of two "numerically equal but opposite in sign" numbers.

Maybe your students do not use it, but in this connection one of my students assured me that these $2(x-3)$'s would "knock each other out." Now, I do not attend the prize fights, but the phrase "knock each other out" brings to my mind two sparsely clad individuals whose hands are encased in boxing gloves wearily pushing each other about the ring until each lands a blow that causes the other to fall prone on the floor. That is not what these $(x-3)$'s do to each other.

I trust that those of you who teach trigonometry are not guilty of *saying cos* x and *tan* x for *cosine* x and *tangent* x, and of using *trig.* for "trigonometry."

My final plea is to those of us who refer to our subjects as math. Does your English teacher teach Lit? Does your science teacher teach zo? Does your history teacher teach polit? Perhaps they do, but that is no excuse for our teaching math. Let us teach mathematics!

On to Baltimore!

Have you made your plans to be in Baltimore at the Annual Meeting of the National Council of Teachers of Mathematics on March 30th. and 31st. and April 1st. and 2nd., 1949?

Headquarters: Lord Baltimore Hotel. Famous leaders you would like to see and hear will be there. See the program on page 104 of this issue.

◆ THE ART OF TEACHING ◆

Presentation of the Locus Problem*

By DOROTHY JOHNSON
Staples, Minnesota

A DIFFICULTY often experienced in teaching the locus problem is that the pupil allocates this unit in his mind as something separate from the rest of the geometry course. That tendency can be counteracted if the pupil (1) gains a clear conception of the definition of the word, and (2) links this unit with his previous experiences.

That the locus of points is the path of a point moving under certain fixed conditions is to us a simple definition. For the pupil, this needs to be clarified. Of course, distances can be measured and loci determined in that way, but a very elementary method can be used which does away with measuring, introduces the idea of motion, and adds interest. Ask the pupil to visualize an inked mechanical dot over which you have control. Then place a point P on the board and command the inked robot to move always two inches from the point P. Instantly even the slowest pupil can actually see the path left by the inked dot as it describes the circle with center P and radius of 2″. Draw the circle with a dotted line. Now place two points on the board and command the inked robot to move always equidistant from the two points. The perpendicular bisector of the line connecting the two points is then drawn with a dotted line. Continue with the other special cases, at this time omitting the case of the locus equidistant from a straight line.

Within a few minutes the entire class can see how the point generates its path under certain fixed conditions. After you have told them that the dotted lines which have been drawn were the loci, they can formulate the definition.

Now that the class understands what a locus is, the problem can be stated. Under each diagram clearly state the question and answer.

At this time, seat work in construction of loci and also practice work in drawing various loci is essential.

Since the locus has been defined, the simple cases have been discussed, and the problem carefully stated, preparation for the logical proof can be made. Review a problem in the graphing of a first degree equation and point out that the graph is the path of a point moving under the conditions set by the equation. Since that is the definition of a locus, the pupil realizes that he has solved the locus problem in algebra. Review the proof of a graph that (1) all points on the graph must satisfy the equation, and (2) that all points that do satisfy the equation lie on the graph. Make it clear to them that since a graph is a locus, the same method of proof is used now.

Intuitively prove that the locus determined 2″ from point O satisfied the above conditions. At this point, discuss the locus of points at stated distance from line AB applying the above statements to obtain the complete locus.

The formal proof can now be given and because of the preliminary work, the pupil better understands why the proof consists in proving (1) every point on the locus

* Presented ot Mathematics Section of C.M.E.A., St. Cloud, Minn., Oct. 17, 1947.

satisfies and (2) every point which does satisfy is on the locus.

To introduce intersection of loci, several illustrations could be used. The home town could be located by use of a road map. For instance follow the horizontal line G until it meets the vertical line 3. G is the locus of points G units from the northern border; 3 is the locus of points 3 units from the western border. The intersection of the loci is the location of the city. The pupil can then see that the intersection of loci determines a point which satisfies the conditions of both loci.

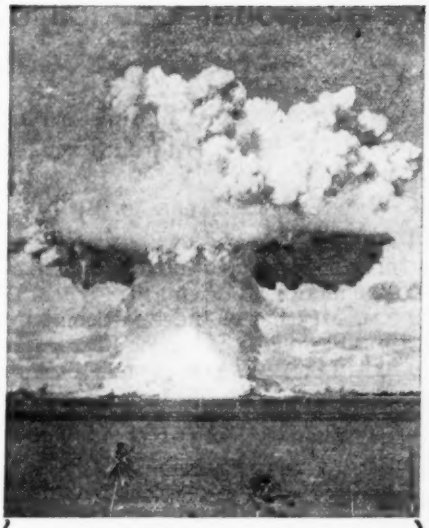
Another illustration which is very important is the graphing of two simultaneous linear equations. Again the pupil sees that the intersection is the location of a point which satisfies the condition of both graphs or loci. By actually reviewing the graphing, the pupil sees that the study of intersection of loci is again something which he has had before.

One does not need to limit oneself to text book problems. Excellent problems on locations of cities by latitude and longitude can be formulated. Air maps can be used to determine distances of the home town from foreign cities and to note our location in relation to the rest of the world.

The locus unit then, instead of standing apart, can be made into one of the most interesting units in the course. It can be linked with the pupil's past mathematics knowledge and from the standpoint of locations of points, can be used as the instrument for teaching global relations.

National Council Yearbooks!

The National Council Yearbooks are rapidly going out of print. Teachers who wish complete files and, particularly, school libraries who lack certain books should order now.—EDITOR.



Once this was just an algebraic formula

Did you know that the atomic bomb with its typical mushroom formation once was just an algebraic formula? With the tremendous energy release over Hiroshima, the atomic age began and the world suddenly realized the significance of $E=mc^2$ [Energy=mass x (speed of light)^2].

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The National Council of Teachers of Mathematics
Twenty-Seventh Annual Meeting

Lord Baltimore Hotel, Baltimore, Maryland, March 30, 31, April 1 and 2, 1949

WEDNESDAY, MARCH 30, 1949

2:00 P.M. Meeting of the Board of Directors of the National Council of Teachers of Mathematics. Parlor M

7:30 P.M. Meeting of the Board of Directors. Parlor M

3:30 to 9:00 P.M. Registration. Mezzanine Floor

THURSDAY, MARCH 31, 1949

8:00 A.M. to 9:00 P.M. Registration. Mezzanine Floor

9:00 A.M. to 12:00 Noon. *Visit Baltimore Schools*

Schools will hold open house for persons attending the NCTM meeting on Thursday, March 31, from 9:00 A.M. to 12:00 Noon. Those interested in visiting any type of school may either send requests in advance to Mrs. Nanette Blackiston, 3 East 25th Street, Baltimore 18, Maryland or secure detailed information at the registration desk on Wednesday, March 30th.

1:30 to 6:00 P.M. Trip to Fort McHenry in Baltimore and the United States Naval Academy at Annapolis, Maryland

Note: Buses leave hotel at 1:00 P.M. Cost $1.00. Reservations should be made in advance by sending check or money order to Miss Margaret Heinzerling, Southern High School, Warren Ave., and William St. Baltimore 30, Maryland. Reservations paid for before March 15 will be acknowledged by mail.

8:00 P.M. First General Meeting. Calvert Ball Room

Get Acquainted with Your Council

In which we hear from State Chairman Kenneth Brown, State Representatives, Delegates of Affiliated Clubs, and Members of the Board of Directors.

Refreshments

Members of the Council are guests of the Baltimore City Mathematics Teachers.

FRIDAY, APRIL 1, 1949

8:00 A.M. to 9:00 P.M. Registration. Mezzanine Floor

9:00 to 11:30 A.M. Mathematics Films. Parlor M

(See list at end of this program)

9:30 to 11:30 A.M. Elementary School Section. Fairmont Room

Presiding: Veryl Schult, Wilson Teachers College, Washington, D. C.

What Mathematics Means to Children in Elementary Schools

Bess Goodykoonst, Director, Division of Elementary Education, U. S. Office of Education, Washington, D. C.

A Meaning Approach to the Teaching of Common Fractions in the Elementary School

Harold E. Moser, State Teachers College, Towson, Maryland

Some Implications for Mathematics Instruction of the National Standardization of Metropolitan Arithmetic Test

Walter N. Durost, former Director of the Test Division, World Book Co., Yonkers, N. Y. Boston University, Boston, Mass.

9:30 to 11:30 A.M. Secondary School Section. Center Calvert Ball Room

Presiding: Marie S. Wilcox, Washington High School, Indianapolis, Indiana

How is Mathematics Being Taught? Results of a National Survey

Henry W. Syer, Boston University, Boston, Massachusetts

Recent Experiments in Tenth and Eleventh Year Mathematics in New York State

Paul J. Smith, East High School, Rochester, New York

The Contribution of Mathematics to the School Safety Program

Mildred Keiffer, Supervising Teacher, Public Schools, Cincinnati, Ohio

A Functional Mathematics Program for Secondary Schools

William A. Gager, University of Florida, Gainesville, Florida

9:30 to 11:30 A.M. Junior College Section. Phoenix Room

Presiding: Virgil S. Mallory, State Teachers College, Montclair, New Jersey

General Mathematics Courses in the Junior College
Kenneth E. Brown, University of Tennessee, Knoxville, Tennessee
Grade Thirteen Mathematics in Ontario
Robert E. K. Rourke, Headmaster, Pickering College, Newmarket, Ontario
Meaningful Applications of Trigonometry
Lyman M. Kells, United States Naval Academy, Annapolis, Maryland

9:30 to 11:30 A.M. In-Service Training Section. Mt. Vernon Room
Presiding: Charles H. Butler, Western Michigan College of Education, Kalamazoo, Michigan
In-Service Training in Los Angeles
Dale Carpenter, Supervisor, Mathematics Education Section, Los Angeles City Board of Education, Los Angeles, California
Our Baltimore In-Service Training Program
Grover W. Norris, Supervisor of Mathematics, Department of Education, Baltimore, Maryland
The Function of a Department Head
C. H. Mergendahl, Newton High School, Newtonville, Massachusetts

10:00 to 11:30 A.M. Discussion and Clinic Sessions. Seventeenth Floor

Note: Reservations for attendance at all discussion sessions should be made in advance by sending first, second, third and fourth choices to Miss Margaret Heinzerling, Southern High School, Warren Ave. and William St., Baltimore 30, Maryland

Group 1. Topic: "What Materials Are Needed for a Meaningful Program in Arithmetic?"
Leader: Foster E. Grossnickle, State Teachers College, Jersey City, New Jersey
Group 2. Topic: "What is a Good Program in General Mathematics?"
Leader: Raleigh Schorling, University of Michigan, Ann Arbor, Michigan
Group 3. Topic: "Teaching Locus"
Leader: A. M. Welchons, Arsenal Technical High School, Indianapolis, Indiana
Group 4. Topic: "How Can We Meet the Challenge of the Present Mathematical Situation?"
Leader: W. Betz, Specialist in Mathematics, Rochester, New York
Group 5. Topic: "What Shall We Include in the Mathematics Course for Seniors?"
Leader: John R. Mayor, University of Wisconsin, Madison, Wisconsin
Group 6. Topic: "What Methods Can Be Used for Creating New Interest in Mathematics?"
Leader: Walter H. Carnahan, D. C. Heath and Company, Boston, Massachusetts

12:00 M. Get-Acquainted Luncheon. Caswell Room

Note: Make reservations in advance. Send $1.75 to Miss Margaret Heinzerling, Southern High School, Baltimore 30, Maryland

Luncheon Speaker: Miss Elisabeth Achelis, President of the World Calendar Association, Inc., New York, N. Y.

1:30 to 5:00 P.M. Mathematics Films. Parlor M
(See List at end of this Program)

2:00 to 4:00 P.M. Elementary School Section. Caswell Room
Presiding: F. Lynwood Wren, George Peabody College for Teachers, Nashville, Tennessee
Of What Value Can a Case Study in Arithmetic Be to a Classroom Teacher?
Ella Marth, Harris Teachers College, St. Louis, Missouri
Arithmetic Readiness and the First Grade Child
Charlotte W. Junge, Wayne University, Detroit, Michigan
Meaningful Long Division—A Development from the Second Grade on
R. R. Smith, Coordinator of Mathematics, Public Schools, Springfield, Massachusetts

2:00 to 4:00 P.M. Applications of Mathematics Section. Center Calvert Ball Room
Presiding: W. W. Rankin, Duke University, Durham, North Carolina
Modern Quality Control (Illustrated with motion pictures)
S. Collier, Director of Quality Control, Johns Manville Corporation, New York, New York
Computation and Its Tools
Ida Rhodes, Mathematician, Machine Development Laboratory, Applied Mathematics Division, National Bureau of Standards, Washington, D. C.

2:00 to 4:00 P.M. Tests and Evaluation Section. Federal Hill Room
Presiding: Raleigh Schorling, University of Michigan, Ann Arbor, Michigan

Duds—in Mathematics
John L. Stenquist, Director, Bureau of Measurement, Statistics and Research, Dept. of Education, Baltimore, Maryland
Tests of Understanding in Secondary Mathematics
Maurice L. Hartung, University of Chicago, Chicago, Illinois
Measuring the Development of Mathematics Ability
Warren G. Findley, Head, Test Construction Department, Educational Testing Service, Princeton, New Jersey.

2:00 to 4:00 P.M. Instructional and Learning Aids Section. Founders Room
Presiding: Martha Hildebrandt, Proviso High School, Maywood, Illinois
New Teaching Aids for Plane Geometry
Amelia Richardson, Technical High School, McKeesport, Pennsylvania
The Design of a Mathematics Laboratory
Lt. Col. Robert C. Yates, United States Military Academy, West Point, New York
Instructional and Learning Aids and Their Use in the Classroom
George R. Anderson, State Teachers College, Millersville, Pennsylvania

2:00 to 4:00 P.M. Teacher Training Section. Phoenix Room
Presiding: Lee Boyer, State Teachers College, Millersville, Pennsylvania
A Study in Needed Redirection in the Training of Teachers of Arithmetic
V. J. Glennon, Syracuse University, Syracuse, New York
Mathematics in General Education
Earle T. Hawkins, President, State Teachers College, Towson, Maryland
Some Needed Revisions of Teacher Training Curricula
H. Van Engen, State Teachers College, Cedar Falls Iowa

2:00 to 4:00 P.M. Junior College Section. Mount Vernon Room
Presiding: E. W. Schreiber, Western Illinois State College, Macomb, Illinois
Visual Aids—Stimuli to Mathematical Research
Joseph Hilsenrath, Scientific Education Advisor, National Bureau of Standards, Washington, D. C.
The Basic Knowledge Requirement in Mathematics at the University of Illinois
Vivian Nuess, University of Illinois, Urbana, Illinois
First the Problem, Then the Mathematics
Albert W. Recht, University of Denver, Denver, Colorado

2:00 to 3:30 P.M. Discussion and Clinic Sessions. Seventeenth Floor

Note: Reservations for attendance at all discussion sections should be made in advance—see note at end of program)

Group 7. Topic: "Learning Arithmetic by Thinking versus Learning by Repetition"
Leader: John R. Clark, Teachers College, Columbia University, NewYork, New York
Group 8. Topic: "Better Living Through Mathematical Understandings"
Leader: Mary C. Rogers, Roosevelt Junior High School, Westfield, New Jersey
Group 9. Topic: "What Must be Done for the Brighter Pupils in Ninth Grade Algebra?"
Leader: Dorothy S. Wheeler, Bulkeley High School, Hartford, Connecticut
Group 10. Topic: "Problems in the Teaching of Geometry"
Leader: Virgil S. Mallory, State Teachers College, Montclair, New Jersey
Group 11. Topic: "How Can We Provide Better Coordination between High School and College Mathematics Programs?"
Leader: H. W. Charlesworth, East High School, Denver, Colorado
Group 12. Topic: "What is General Mathematics in the Junior College?"
Leader: Charles H. Butler, Western Michigan College of Education, Kalamazoo, Michigan
Group 13. Topic: "Diagnostic and Remedial Procedures in Arithmetic for the High School"
Leader: Nathan Lazar, Midwood High School, Brooklyn, New York and Teachers College, Columbia University, New York City

4:00 to 5:00 P.M. Reception and Tea. Chinese Suite
Members of the Council are Guests of the Mathematics Section of the Maryland Teachers Association

6:00 P.M. Dinner—Fawcett's Ohio Staters
Mrs. Clarence E. Hardgrove, Dept. of Education, Ohio State University, Columbus 10, Ohio, in charge.

7:00 P.M. Film—"Life at Denver University," Calvert Ball Room

8:00 P.M. General Meeting. Calvert Ball Room

Presiding: E. H. C. Hildebrandt, Northwestern University, Evanston, Illinois

Welcome to Baltimore
William H. Lemmel, Superintendent of Public Instruction, Baltimore, Maryland

Welcome to Maryland
Thomas G. Pullen, Jr., State Superintendent of Schools, Baltimore, Maryland

Mathematics and Modern Educational Trends
Carroll V. Newsom, Assistant Commissioner for Higher Education for the State of New York, Albany, New York

Human Relations in Technical Times
Dwayne Orton, Director of Education, International Business Machines Corporation, New York, New York.

SATURDAY, APRIL 2, 1949

7:00 to 8:30 A.M. Breakfasts

1. Duke Mathematics Institute Reunion Breakfast, $1.50
   Elizabeth Gardner, Baltimore, Md., in charge
2. "Reeve Gang" Reunion Breakfast. $1.50
   Kenneth Brown, University of Tennessee, Knoxville, Tennessee, in charge

8:00 A.M. to 3 00 P.M. Registration. Mezzanine Floor

8:30 to 9:30 A.M. General Business Meeting of the National Council of Teachers of Mathematics. Fairmont Room

9:30 A.M. to 12:00 Noon. Films. Parlor M (See list at end of this Program)

10:00 A.M. to Noon. Elementary School Section. Phoenix Room

Presiding: Mary Adams, Assistant Superintendent (in charge of elementary schools), Department of Education, Baltimore, Maryland

Topic: Initiating a Program of Meaningful Arithmetic in the Elementary Grades:

a) The Formation of an Arithmetic Committee
b) Building a Background of Information with the Committee
c) In-Service Training of Teachers

Sarah M. Reese and Audrey Deppenbrock, Public Schools of Baltimore

10:00 A.M. to Noon. Junior High School Section. Caswell Room

Presiding: Nanette Blackiston, Supervisor of Junior High School Mathematics, Department of Education, Baltimore, Maryland

Dramatizing Junior Mathematics
Julia E. Diggins, Paul Junior High School, Washington, D. C.

Let's Hear from the Students
Edith Woolsey, Sanford Junior High School, Minneapolis, Minnesota

Visual Aids for the Teaching of Geometry in the Junior High Grades
H. v. Baravalle, Adelphi College, Garden City, New York

Mathematics Through the Eye-Gate
Wilmer F. Bennett, Langley Junior High School, Washington, D. C.

10:00 A.M. to Noon. Senior High School Section. Center Calvert Ball Room

Presiding: Walter Carnahan, D. C. Heath and Co., Boston, Massachusetts

Algebra, A Generalized Arithmetic
Jean Tully Stockton College, Stockton, California

What a Teacher Expects from Supervision
Catherine A. V. Lyons, Veterans' Training Program, Oliver High School, Pittsburgh, Pennsylvania

Mathematical Understandings and Judgments Retained by College Freshmen
Ben A. Sueltz, State Teachers College, Cortland, New York

10:00 A.M. to Noon. Junior College Section. Federal Hill Room

Presiding: George Hawkins, Lyons Township High School and Junior College, LaGrange, Illinois

The Status of Teaching Loads, Salaries and Preparation of Junior College Mathematics Teachers
H. G. Ayre, Western Illinois State College, Macomb, Illinois

Freshman Mathematics for General Students in Arts Colleges
Howard E. Wahlert, New York University, New York, N. Y.

Visual Aids—Stimuli to Mathematical Research
Joseph Hilsenrath, Scientific Education Advisor, National Bureau of Standards, Washington, D. C.

10:00 A.M. to Noon. Pre-Engineering Mathematics Section. Founders Room

Presiding: James H. Zant, Oklahoma A. and M. College, Stillwater, Oklahoma

The Relations of Secondary Mathe-

matics to Engineering Education
Dean S. S. Steinberg, College of Engineering, University of Maryland, College Park, Maryland
The American Society for Engineering Education Committee on Secondary Schools
Henry H. Armsby, Specialist in Engineering Education, U. S. Office of Education, Washington, D. C.
The Response of the Secondary School to the Problem of Engineering Mathematics
A Discussion led by C. N. Shuster, State Teachers College, Trenton, New Jersey

10:00 A.M. Discussion and Clinic Groups. Seventeenth Floor

Note: Reservation for attendance at a discussion section should be made in advance—see note at end of program

Group 14. Topic: "Arithmetic Teaching Techniques"
Leader: Joseph J. Urbancek, Chicago Teachers College, Chicago, Illinois
Group 15. Topic: "T. A. V. Aids (Tactile-Audio-Visual Aids) Help Numerical Communications"
Leader: Amanda Loughren, Supervisor, Department of Mathematics, Board of Education, Elizabeth, New Jersey
Group 16. Topic: "Will the Present Tendency to Omit Proofs from High School Geometry Tests Harm the Teaching of Geometry?"
Leader: Ona Kraft, Collinwood High School, Cleveland, Ohio
Group 17. Topic: "Adding Reality to Mathematics Instruction"
Leader: Kenneth P. Kidd, University of Florida, Gainesville, Florida
Group 18. Topic: "Work Books, Their Use and Abuse!"
Leader: Marie S. Wilcox, Washington High School, Indianapolis, Indiana
Group 19. Topic: "How Can the College Professors and Campus Training School Supervisors Work Together for Preparing Mathematics Teachers?"
Leader: Velma Cloyd, East Tennessee State College, Johnson City, Tennessee
Group 20. Topic: "The Slow Learning Child: His Characteristics, His Limitations, His Possibilities"
Leader: Mary A. Potter, Supervisor, Public Schools, Racine, Wisconsin
Group 21. Topic: "The Senior High School Mathematics for Better Students"
Leader: Howard F. Fehr, Teachers College, Columbia University, New York, New York
Group 22. Topic: "What Simple Applications of Algebra Should be Used to Coordinate the Teaching of Algebra with Arithmetic?"
Leader: Lee E. Boyer, State Teachers College, Millersville, Pennsylvania
Group 23. Topic: "The Construction and Use of Models in a Mathematics Laboratory"
Leader: Lt. Col. Robert C. Yates, United States Military Academy, West Point, New York
Group 24. Topic: "The Function of the Parents in Homework"
Leader: Mildred Keiffer, Supervising Teacher, Public Schools, Cincinnati, Ohio
Group 25. Topic: "Materials and Devices for an Experimental Approach to Statistics"
Leader: Paul C. Clifford, State Teachers College, Montclair, New Jersey
Group 26. Topic: "How Can We Use Instructional Aids More Effectively in Our Teaching of High School Mathematics?"
Leader: Madeline Messner, High School, Roselle, New Jersey

1:00 to 4:30 P.M. Mathematics Films. Parlor M
(See list at end of this program)

2:00 P.M. Elementary School Section. Center Calvert Ball Room
Presiding: Vera Sanford, State Teachers College, Oneonta, New York
What Constitutes Remedial Work in Arithmetic?
Lesta Hoel, Supervisor of Mathematics, Public Schools, Portland, Oregon
Meaningful Learning of the Operations With Fractions
Lenore John, The Laboratory School, The University of Chicago, Chicago, Illinois
Arithmetic—Tool Subject or Content Subject
W. A. Brownell, Chairman, Department of Education, Duke University, Durham, North Carolina.

2:00 P.M. General Mathematics Section. Caswell Room
Presiding: Mary A. Potter, Supervisor of Mathematics, Public Schools, Racine, Wisconsin
Selling General Mathematics
Allene Archer, Thomas Jefferson High School, Richmond, Virginia

General Mathematics—A Must for the High School
Eugenie C. Hausle, James Monroe High School, New York, New York
Do Slow Learners Need Special Treatment in General Mathematics?
F. G. Lankford, University of Virginia, Charlottesville, Virginia

2:00 P.M. Applications Section. Phoenix Room
Presiding: H. W. Charlesworth, East High School, Denver, Colorado
Applications of Mathematics in the Curricula of Schools of Nursing
K. Virginia Betzold, Associate Director of the School of Nursing, The Johns Hopkins Hospital, Baltimore, Maryland
Some Applications of Mathematics in a Chemical Company
R. S. Spencer, Physical Research Laboratory, Dow Chemical Company, Midland, Michigan
The Statistical Tools that an Engineer Uses
Paul Clifford, State Teachers College, Montclair, New Jersey

2:00 P.M. College Preparatory Section. Founders Room
Presiding: R. R. Smith, Coordinator of Mathematics, Public Schools, Springfield, Massachusetts
A New Challenge to the Secondary Mathematics Curriculum
Edwin C. Douglas, Taft School, Watertown, Connecticut
How to Prepare for Program 2 of the CEEB
J. B. Adkins, Phillips Exeter Academy, Exeter, New Hampshire
Changes in Content and Method Taking Place in Secondary School Mathematics—and Their Implications
W. S. Litterick, Acting Headmaster, The Peddie School, Hightstown, New Jersey

2:00 P.M. Teacher Training Section. Federal Hill Room
Presiding: Carl N. Shuster, State Teachers College, Trenton, New Jersey
Laboratory Experiences to Enrich the Pre-Service Training Program for Teachers of Mathematics in the Secondary Schools
Ida May Bernhard, Public Schools—Demonstration School, Southwest Texas State Teachers College, San Marcos, Texas
Teacher Preparation for a New Era in Mathematics
Daniel W. Snader, University of Illinois at Galesburg, Illinois
The Certification of Teachers of Mathematics
W. I. Layton, Alabama Polytechnic Institute, Auburn, Alabama

2:00 P.M. Discussion Groups and Clinics. Seventeenth Floor
Note: Reservations for attendance at all discussion sections should be made in advance. See note at end of program
Group 27. Topic: "Basic Meanings in Arithmetic"
Leader: F. Lynwood Wren, George Peabody College for Teachers, Nashville, Tennessee
Group 28. Topic: "How and Why should We Emphasize Basic Generalizations in Teaching Arithmetic?"
Leader: Irene Sauble, Director, Exact Sciences, Detroit Public Schools, Detroit, Michigan
Group 29. Topic: "What Should Be the Content of a General Mathematics Course at 7th and 8th Grade Level? How Should this Content Be Organized in Terms of Sequence?"
Leader: John J. Kinsella, Ohio State University, Columbus, Ohio
Group 30. Topic: "Algemathics Through Graphicmathics: A Better Second Track Program for the General Mathematics Student"
Leader: Elenore M. Lazansky, Lafayette High School, Lafayette, California
Group 31. Topic: "How Can We Adapt Instruction to the Individual Abilities of Pupils?"
Leader: George Hawkins, Lyons Township High School and Junior College, La Grange, Illinois
Group 32. Topic: "Is Mathematics Serving the Needs of Our Communities?"
Leader: Veryl Schult, Wilson Teachers College, Washington, D. C.
Group 33. Topic: "The Mathematics Curriculum in the Small High School"
Leader: H. Vernon Price, State University of Iowa, Iowa City, Iowa
Group 34. Topic: "How Can we Provide Better Coordination between Our High School and College Programs?"
Leader: Florence E. Loose, Wilmington High School, Wilmington, Delaware
Group 35. Topic: "Can We Build an Effective Mathematics Laboratory Around Low Cost Student or Teacher Made Devices?"
Leader: Joseph Hilsenrath, Scientific Education Advisor, National Bureau of Standards, Washington, D. C.

Group 36. Topic: "Aids in the Study of Algebra"
Leader: W. W. Rankin, Duke University, Durham, North Carolina
Group 37. Topic: "How Can Films and Filmstrips Be Used Most Effectively in the Teaching of Algebra and Geometry?"
Leader: Donovan Johnson, University High School, University of Minnesota, Minneapolis
Group 38. Topic: "The Role of Geometry in General Education"
Leader: Harold Fawcett, Chairman of the Department of Education, Ohio State University, Columbus, Ohio
Group 39. Topic: "What Are the Best Methods of Motivating Students of Low Ability?"
Leader: M. L. Hartung, University of Chicago, Chicago, Illinois
Group 40. Topic: "The Relation of Secondary Mathematics to Engineering Education"
Leader: James H. Zant, Oklahoma A. and M. College, Stillwater, Oklahoma
Group 41. Topic: "What Type of Mathematics Should Be Included in the General Mathematics Courses in College?"
Leader: Phillip S. Jones, University of Michigan, Ann Arbor, Michigan

6:30 P.M. Annual Banquet. Calvert Ball Room
Address
Detlev W. Bronk, President, The Johns Hopkins University, Baltimore, Maryland

ANNOUNCEMENTS

Room Reservations. All applications for rooms should be sent directly to the Lord Baltimore Hotel, Baltimore 30, Maryland. A form, similar to that shown in this program announcement, may be used.

Banquet. The banquet will be held on Saturday evening, April 2 at 6:30 P.M. in the Calvert Ball Room of the Lord Baltimore Hotel. Reservations may be made by sending $3.75 in advance to Miss Margaret Heinzerling, and, if made by March 15th, will be acknowledged by mail.

Discussion Groups and Clinics. It will be necessary to limit the number in attendance at these group meetings. It is therefore recommended that first, second, third and fourth choices be sent to Miss Heinzerling as soon as possible. Admittance cards will be returned by mail to those making their requests before March 15th. Additional opportunities to register for a discussion group and obtain an admittance card may be arranged at the registration desk at the time of the convention. Admittance to these groups will be by admission card only.

Exhibits. There will be an exhibit of mathematical models, instruments and teaching aids on the Mezzanine Floor. Teachers are invited to bring materials for the exhibit and should communicate with Mr. Alfred E. Culley, School No. 406, Chatham and Eldorado Ave., Baltimore 7, Maryland regarding exhibit space.

Commerical Exhibits. Textbooks and commercial teaching aids will be on exhibit on the Mezzanine Floor. Inquiries for exhibit space should be addressed to Mr. George Lundberg, School No. 91, Hilton St. and Morley Ave., Baltimore 29, Maryland.

Films and Filmstrips. Some of the latest mathematics films and filmstrips will be shown on mornings and afternoons of Friday and Saturday in Parlor M on the third floor. Persons interested in viewing films listed below, may forward requests to Miss Jaclyn Rivkin, School No. 70 Annex, Baltimore, Maryland. Titles to be shown will be chosen from the following:

Films: Arithmetic: *Let's Count, We Discover Fractions, Per Cent in Everyday Life, Meaning of Long Division, What is Four?, Parts of Nine, Parts of Things, Introduction to Fractions, Meaning of Percentage, Individual Differences in Arithmetic, Meaning of Long Division.*

Social Arithmetic and General Mathematics: *What is Money, Fred Meets a Bank, Banks and Credit, Using the Bank, Your Thrift Habits, Sharing Economic Risks, Installment Buying, Check and Double Check, Money at Work, Credit Unions—John Doe's Bank, Search for Security, The Benefactor, Story of Money, Managing the Family Income, Your Dollars in Uniform, Property Taxation, Protecting the Customer, Federal Taxation, The Work of the Stock Exchange, The Language of Graphs, The Micrometer, Measurement.*

Algebra: *Algebra in Everyday Life, The Slide Rule.*

Geometry: *Geometry and You, Practical Geometry, Angles, Areas, Chords and Tangents, Locus, Polygons, Properties of Triangles, Behind the Shop Drawing, Pythagorean Theorem.*

Filmstrips: Arithmetic: *What Numbers Mean, Zero a Place Holder, A Number Family in Addition, Compound Subtraction, The Two in Division, Addition and Subtraction of Fractions, Comparing Frac-*

tions, Improper Fractions, Changing Fractions to a Common Denominator, Multiplication and Division of Fractions, Counting to 10, Counting from 40 to 100, Writing Numbers to 100, Reading Numbers to 100, Working with Numbers to 100, Square Root and Cube Root, Order of Operations.

Algebra: *Introduction to Algebra, Introduction to Signed Numbers, Origin of Algebra, Basic Definitions of Algebra, Positive and Negative Numbers, Exponents and Logarithms, Problem Analysis, Plotting Graphs.*

Geometry: *Introduction to Plane Geometry, Locus, Geometry in Art, Deductive Reasoning, Mistakes in Thinking, Angular Measurement, Constructions, Basic Angles and Experimental Geometry, Parallel Lines and Transversals, Congruent and Overlapping Triangles, Introduction to Circles.*

COMMITTEES

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Walter H. Carnahan, D. C. Heath and Company, Boston, Massachusetts
H. W. Charlesworth, East High School, Denver, Colorado
Agnes Herbert, Clifton Park Junior High School, Baltimore, Maryland
Mary A. Potter, Public Schools, Racine, Wisconsin
Veryl Schult, Wilson Teachers College, Washington, D. C.
Dorothy Wheeler, Bulkeley High School, Hartford, Connecticut
James H. Zant, Oklahoma A. and M. College, Stillwater, Oklahoma

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(Members are from Baltimore schools unless otherwise indicated)

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ROOM RESERVATION INFORMATION

All applications for rooms in the Lord Baltimore Hotel should be sent directly to the hotel. Rates for rooms are as follows:

Single Rooms (for one) $4.00, $4.25, $4.50, $4.75, $5.00, $5.25, $6.00, $6.50, $7.50, $8.00
Double Rooms (for two) $6.50, $7.00, $7.50, $8.50, $9.00, $10,00, $10.50
Twin Bedrooms (for two) $7.50, $8.00, $8.50, $9.00, $10.00, $10.50
Suites (Parlor, Bedroom and two Baths) (for one or two) $18.00
3 in a room, $2.00 additional charge for 3rd occupant.
The following form may be used in writing for reservations:

Lord Baltimore Hotel
Baltimore 3, Maryland

Gentlemen:
Please reserve _____
(Type of room and rate desired)
at the time of the convention of the National Council of Teachers of Mathematics, March 30th–April 2, 1949. I will arrive in Baltimore

_____ at _____ A.M. P.M.
(Date) (Time)

Name _____

Address _____

City _____

Reservations for Annapolis trip, Get-Acquainted Luncheon, Banquet, and for discussion groups should be made as follows:

Miss Margaret Heinzerling
Southern High School
Warren Avenue and William Street
Baltimore 30, Maryland

Please make the following reservations:

_____ Reservation(s) for Thursday's Tour to Annapolis at $1.00

_____ Reservation(s) for Friday's Get-Acquainted Luncheon at $1.75

_____ Reservation(s) for Saturday's Banquet at $3.75

I inclose check / money order for $_____.

My choices for Discussion Groups are as follows:

Friday A.M. 1st Choice, Group No. _____; 2nd Choice, Group No. _____

Friday P.M. 1st Choice, Group No. _____; 2nd Choice, Group No. _____

Saturday A.M. 1st Choice, Group No. _____; 2nd Choice, Group No. _____

3rd Choice, Group No. _____; 4th Choice, Group No. _____

Saturday P.M. 1st Choice, Group No. _____; 2nd Choice, Group No. _____

3rd Choice, Group No. _____; 4th Choice, Group No. _____

Name _____

Address _____

City _____

ALGEBRA: *First Course*

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Just published, a new textbook which emphasizes understanding of the languages and concepts of algebra, geared to the learner by ingenious use of inductive thinking and realistic applications. Leads concretely from arithmetic to algebra, stresses proficiency in basic skills, provides a solid foundation for a study of more advanced mathematics.

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AIDS TO TEACHING

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DONOVAN A. JOHNSON
College of Education
University of Minnesota
Minneapolis, Minnesota

Note: The manufacturers have informed us that there is a price change on Slidecraft Material (SL.4—Oct. 1948). They also tell us that other larger transparent plastic materials are now available for use on the overhead projector. Check with them for further details.

BOOKLETS

B.16—*How Long is a Rod?*

Johansson Division, Ford Motor Company, 3000 Schaefer Road, Dearborn, Michigan.

Leaflet; 6″×9″, six pages; Free.

Description: This folder in color contains one large and five small pictures concerning the history of the measurement of length. They show the method of determining the cubit, digit, palm, inch, foot, and yard, with an accompanying description of the procedures used by the Egyptians, Greeks, English, and Americans. The description ends by mentioning Johansson Blocks, and one of the six pages is devoted to explaining them.

Evaluation: Five of the six pages of this leaflet are extremely interesting, helpful, and useful for project work and bulletin board display. Few pictures reconstructing early mathematical instruments and methods are available. When they are as attractive as these they should have immense and widespread use by teachers from grades 5 through 12. The Ford Motor Company has some copies of this leaflet still available, and it is hoped that interested teachers will persuade them to continue reprinting them for a long time in the future.

B.17—*Mathematics Visual and Teaching Aids*

Teaching Aids Service, New Jersey State Teachers College, Upper Montclair, New Jersey

Booklet; 8½″×11″, 30 pages; 1947; 75¢

Description: This booklet contains a very complete list of charts and maps, devices, exhibits, films, slides and filmslides, games, pictures, publications and recordings, and was compiled by Frances Goon, Charles W. Martin and John J. O'Brien, Jr. from materials collected by Dr. Lili Heimers, director of the Teaching Aids Service. It is divided into six chapters, giving general information, information on arithmetic, algebra, geometry, trigonometry, and advanced mathematics. Under each of these headings the aids are divided into their various types. Each aid is followed by a short description, as well as source and price.

Evaluation: The completeness of this list is outstanding, but no mention is made of the date at which it was checked for availability, so it cannot be certain whether the list is sufficiently current. Most descriptions are sufficient, but enough are sketchy and vague to make the reader wish that more space for evaluation had been available. However, this is a very valuable booklet for each school system or even each teacher to own.

B.18—*The Dozen System*

Longmans Green and Co., 55 Fifth Avenue, New York, N. Y.

Booklet; $9\frac{1}{2}'' \times 11\frac{1}{2}''$, 53 pages; 50¢.

Description: George S. Terry, author of "The Dozen System," is an ardent advocate of replacing the decimal system by one with the base twelve. The advantages of a duodecimal system are real but the feasibility of changing bases is questionable. Terry introduces his argument by indicating three advantages of the larger base—an increased number of integral factors, the smallest number with four divisions, and simplified operations. The remainder of the booklet is primarily concerned with the methods of performing simple operations with numbers in the base twelve and with the commercial benefits of such computation.

Evaluation: The treatise is well organized and printed. Its format is superior in every respect and the durable red cover should prolong its usefulness.

Number systems with bases other than ten are discussed in many mathematics classes but seldom with any more intensity than a brief mention of the resulting effects. Since such work is not afforded an important position in the curriculum, students are left curious and unsatisfied. It is for the benefit of these students that *The Dozen System* should be added to the classroom library. The author presents his argument clearly and the operations simply; nevertheless, the concepts are somewhat difficult for the secondary student to grasp and will be most appreciated by the superior mathematics students. Much can be learned about the decimal system through study of another base. This is true because the fundamental operations are learned by rote in the lower grades before the student is capable of fully understanding the concepts involved. By the time he is sufficiently mature to understand, the processes have become habitual and he fails to see the basic ideas involved. Working with another system of numbers removes the veil of habit.

A single copy of the booklet is sufficient for the classroom library; fifty cents is a reasonable price to pay. (Reviewed by Bernard Singer, Boston, Massachusetts.)

CHARTS

C.6—*Nuclear Physics Charts*

Westinghouse School Service, Westinghouse Electric Corp., 306 Fourth Avenue, P.O. Box 1017, Pittsburgh 30, Pennsylvania.

Set of six charts; each $25'' \times 37''$; two colors; $1.00

Descriptive booklet; $6'' \times 9''$; 36 pages; 10¢

Description: The six magnificently planned and executed charts describing nuclear physics may be slightly outside the immediate interest of mathematics classes, but their attractiveness and modernity make it imperative that all possible uses be capitalized by mathematics as well as science teachers. The charts illustrate very fully the following topics: particles of nuclear physics, structure of the nucleus, nuclear reactions, tools of the nuclear physicist, using nuclear energy, progress of nuclear physics. The booklet illustrates each chart in miniature and adds considerable technical detail.

Evaluation: It should be noted that these are not posters propagandizing ideas of nuclear physics, but complete, complicated summaries of a very large number of facts about the subject. Their completeness is frightening at first and their full meaning cannot be grasped below the college level. For teachers and pupils with courage to approach such important new developments and with the attitude that they will learn all they can, they have some usefulness in mathematics classes. There are graphs to be read, examples of plus and minus numbers, and illustration of the equation $E = Mc^2$, ex-

amples of the exponential notation for very small numbers, and the usefulness of subscripts and superscripts in chemical reactions as well as mathematical equations. Not an easy task for ordinary classes, but rewarding to those who can make the time available for study. The present emphasis and obvious importance of nuclear power demands that schools approach this subject and add slight bits to the background of pupils whenever and in whatever subject the opportunity can arise.

FILMS

F.30—*The Isograph*

Bell Telephone Laboratories, 463 West Street, New York 14, N.Y.

16 mm silent film; 1 reel, 400 feet, 15 minutes; black and white; Free (except for postage and insurance).

Description: This film is accompanied by 13 lantern slides, a 14-page mimeographed description of the slides, and a picture of the slides. The film and slides together describe a computing instrument designed at the Bell Telephone Laboratories for finding complex roots of polynomial equations. This is accomplished by a series of cams which are set by operators who then allow the machine to run until approximations of the desired roots are obtained. The mathematical theory necessary to understanding completely the operation involves trigonometric functions, Fourier series, and a bit of functions of a complex variable. The slides and film together form a very complete and clear explanation of this mechanism.

Evaluation: The mathematics involved makes this film suitable for advanced college classes in mathematics, and much of interest and information will be available to them. Strangely enough, however, it is also interesting to high school students. A great deal of their interest, naturally, comes from the awing spectacle of a machine solving mathematical equations. However, those who have studied 12th year trigonometry will find part of the explanation completely within their understanding, and will be pleased to discover that combinations of sines and cosines have some industrial engineering uses. The nature of this material probably makes it more suitable for mathematics clubs, assemblies for mathematics students only, and groups of teachers who wish an evening of mathematics discussion.

INSTRUMENTS

I.7—*Plane Table and Alidade*

Yoder Instruments, East Palestine, Ohio

Surveying Instruments; $16'' \times 21'' \times \frac{3}{4}''$; $26.00

Description: The plane table itself consists of a $16'' \times 21'' \times \frac{3}{4}''$ white pine board which is strengthened by means of tongue and groove construction. It is stained and waxed to make it weather-resistant. A machined bronze plate is attached to the underside of the board and a French joint enables it to be firmly attached to the tripod. Fifty-two-inch ash legs are protected by metal shields at their bases. A steel plumb bob is suspended by tarred fish line from the tripod head.

The case bronze alidade is essentially a ten-inch rule upon whose ends sights are mounted. The sights are bronze strips 3″ high with vertical slots through which the object may be observed. Lines of direction are drawn on paper attached to the plane table as the observations are made.

Evaluation: The plane table and alidade are practical instruments for drawing small-area maps. They are used in the mathematics class to illustrate how actual maps are constructed, to develp ability to construct and interpret scale drawings and to illustrate a very practical application of the proposition that two straight lines intersect in a point.

The instruments may be easily mastered by junior high school students who will appreciate the fact that they are able to simulate processes used in the construc-

tion of government maps. A manual of directions concerning principles of the plane table is supplied with each instrument purchased.

Although the price of the instrument is not extremely high, there is little excuse for purchasing it if the school in question has a carpentry shop. The table and alidade may easily be made at a cost of a dollar or two. (Reviewed by Bernard Singer)

I.8—*Hoot-Nanny*

Northern Signal Company, Inc., 120 North Broadway, Milwaukee, Wisconsin

Device for drawing circular patterns; $3.50

Description: The Hoot-Nanny consists of a mechanical drawing device, paper discs for drawing, six colored pencils, and complete instruction for creating the designs. The device itself is a circular table which turns by means of a geared crank at the side so that a pencil, suspended over the moving circular piece of paper on a linkage arrangement, traces designs which are symmetric with regard to the point of rotation of the table. These designs are traced on circles of paper $5\frac{3}{4}$ inches in diameter, and the booklet claims that 22,000,000 different patterns may be drawn. These vary from simple daisy-like patterns to complicated inter-weaving scrolls which resemble Italian lacework.

Evaluation: The facination of creating new designs by changing the many mechanical settings of the arms and gears of this device will lead to hours of experimentation. In the low school grades it may serve only as a drawing toy to accentuate the beauty and usefulness of symmetric geometric designs. In secondary mathematics, the connections between various settings and resulting types of pattern may be discussed. The apparent randomness of the tracing leads to considerable wonder when the second and third revolutions result in exactly the same pattern. Pupils have expressed an appreciation of the cause-and-effect relationship between the initial setting and completed design, which is recognized as mathematical in character. College classes in mathematics may wish to attempt expressions of the equations of these curves in polar coordinates, but such an undertaking requires considerable analysis.

MODELS

M.3—*The $(a+b)^3$ Cube Set*

The Mathaids Company, 204 Forman Avenue, Syracuse, N.Y.

(See description below); $1.50

Description: The set is composed of eight hardwood blocks which may be assembled to form a four-inch cube. Disassembly of the cube shows that its component parts are one 3-inch cube, three 1-inch×1-inch×3-inch square prisms, three 1-inch×3-inch×3-inch square prisms and one 1-inch cube.

The blocks are accurately cut and the hardness of the black walnut blocks makes the set quite durable.

Evaluation: The set is designed for demonstrating expansion of the binomial $(a+b)^3$ into $a^3+3a^2b+3ab^2+b^3$. The particular dimensions of the blocks serve to show that $(3+1)^3=3^3+3(3)^2(1)+3(3)(1)^2+1^3$. This concrete method of illustration is not new but is convincing nevertheless. It is certainly suited to the understanding and interest of high school students.

One dollar and a half is a reasonable price for the set although it may be made more cheaply in the school carpentry shop. Teaching value of the model will be increased if it is circulated among the students for individual study to supplement its value as a demonstration device. (Reviewed by Bernard Singer)

M.4—*Nautilus Pompilus*

Ward's Natural Science Est., 3000 Ridge Road, Rochester 9, New York.

Shell; 8″ diameter; $4.25

Description: The Nautilus pompilus specimen is a marine mollusk's spiral shell; it is approximately eight inches in diameter. An entire specimen or a half shell may be purchased; the latter form is desirable because it better illustrates the mathematics involved.

A more economical investment may be made in purchasing the Cephalopod Spirula peronii which has a beautiful spiral shape. It is slightly over an inch and one half in diameter and sells for $1.00.

Evaluation: The Nautilus pompilus and Cephalopod Spirula peronii would be of little mathematical significance if they were used only to illustrate the geometric spiral. Their appropriateness in the mathematics classroom is based upon more subtle properties.

"It may not seem possible that a compound interest table, the shell of the chambered nautilus, a particle of radium, and the population of the United States possess a common property which is of significant interest . . . , yet such is the case. Each exemplifies a general law of growth or decay, a law which is inherent in the fact that certain varying magnitudes change at a rate directly proportional to the magnitude itself. . . . " (Elmer B. Mode, "The Elements of Statistics." New York, Prentice-Hall, Inc., 1946, p. 265.) This law may be represented by the equation: $y = AC^{Bx}$.

Use of these shells may be made in introducing secondary students to mathematical phenomena in nature. The nautilus may be used to introduce such topics as the path of planets, the trajectory of projectiles, shape of a suspended chain, and parabolic and elliptical properties in reflecting light, heat, and sound.

The shells are quite expensive due to the fact that they were transported here from the Philippine Islands. The Cephalopod Spirula peronii is not excessively expensive. (Reviewed by Bernard Singer)

MATERIAL FOR LABORATORY WORK

SL.6—*Dilecto Bakelite Fiber Sheets*

Continental-Diamond Fibre Company, Newark, Delaware

Fiber sheets; (See description and prices below.)

Description: The following information has been received from Professor M. H. Ahrendt, of Anderson College, Indiana.

"Laminated plastic fibers are made with various bases, such as canvas, paper, fiberglas, nylon, etc. The type I have used has a canvas base, and is listed by the Continental-Diamond Fibre Company as Grade C. It is available in two colors, natural or black. It is available in thicknesses from .025 inch to 10 inches. The material I have used is .051 inches thick. This material is tough and rigid, and is waterproof. It can be cut with heavy scissors, drilled with a hand drill or punched with a heavy punch, and the corners and edges can be smoothed with sandpaper. Greater thicknesses can be drilled, sawed, machined, tapped, etc.

"The company could not quote prices, since the price depends upon the thickness of the material, the way it is cut, and the like. The material can be bought in sheets, or will be cut into any width strips specified. The minimum charge for one order is $3.00. A $3.00 order, for example, should provide enough strips for making several dozen models of linkages.

"Since this material is so adaptable to working with hand tools, I consider it the best material I have seen for constructing linkages and other types of models that require plane pieces."

Evaluation: Anyone who saw the excellent linkages which Professor Ahrendt had on display at the Indianapolis convention will agree that this material is ideal for model construction. He has pointed out its advantages in his description, quoted above.

EDITORIAL

Improvement of Guidance

THE following quotation appeared in *The Atlantic Monthly* for October 1948 (pages 51–52) in an article by James B. Conant, president of Harvard University and is reproduced here by permission of the publishers of the *Monthly*.

> The improvement of the guidance system may or may not require increased funds. Certainly a better integration of guidance with the teaching should be possible in many schools without much increase in cost. This question of guidance is central to the whole philosophy of a democratic school system which endeavors to make society more fluid.
>
> Guidance is the keystone of the art of public education. At every stage of the educational process, the capabilities of each student should be assessed. Wise counseling should assist the pupil in taking the next step in the educational journey. Eventually each youth should find a satisfying employment corresponding to his or her ambitions and abilities. In many high schools the potential professional talent suffers the most from the present inadequacies. I wish some organization identified in the public mind with concern for *all* American youth would take some dramatic action to demonstrate a vigorous interest in the gifted boy or girl. This would serve as an encouragement to all teachers. The schools would be stimulated in a direction which in some quarters has been rather spurned as being undemocratic and old-fashioned. A National Commission for the Identification of Talented Youth has been suggested by one group of educators; the sponsoring of this by public school administrators and teachers would be the sort of thing I have in mind.

This is a most significant comment and all of the various teacher groups should begin to make plans to improve the guidance situation in their respective subjects. In so far as mathematics is concerned, we are lucky in that the Commission on Post-War Plans has furnished us with an excellent guidance pamphlet in mathematics[1] for use in the schools. As stated before in this magazine each ninth grade pupil should have one of these pamphlets placed in his hands so that he, his teacher, and his parents can better plan his future work. We have already distributed over 25,000 of these pamphlets, but there are still many schools and pupils who have not been supplied with them.

Interested teachers of mathematics should see to it that copies of these pamphlets be placed on sale at all meetings of mathematics teachers local, state or national and THE MATHEMATICS TEACHER will be glad to send out any number of pamphlets on consignment to interested groups. Just let us know how many copies you need; we will ship them to you, and you pay for what you sell and return the rest to THE MATHEMATICS TEACHER.

Reference has already been made in the November issue of THE MATHEMATICS TEACHER to another pamphlet on Guidance called "A Mathematics Student—To Be Or Not To Be?" by a committee of the Michigan section of the *Mathematical Association of America*[2] of which Professor C. Rictmeyer of the Central Michigan College of Education at Mt. Pleasant, Michigan was chairman.

These pamphlets and others that no doubt will follow indicate not only interest in guidance but the great importance of it in the schools.—W.D.R.

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[1] Copies of this pamphlet may be secured for 25¢ each postpaid or in orders of 10 or more copies for 10¢ each postpaid by writing to THE MATHEMATICS TEACHER, 525 W. 120th St., New York 27, N. Y.

[2] Two copies of this pamphlet, one separate copy of the chart (See the frontispiece of this issue) which was its frontispiece, and a letter were mailed by the Committee to every high school principal in Michigan. The chart is reproduced in this issue by permission of Professor Richtmeyer.—W.D.R.

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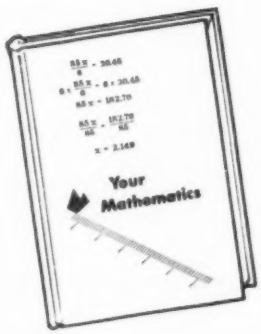

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